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A
COMPLEAT THEORY
OF THE
CONSTRUCTION AND PROPERTIES
OF
VESSELS
WITH
PRACTICAL CONCLUSIONS
FOR THE
MANAGEMENT OF SHIPS
MADE EASY TO NAVIGATORS.

TRANSLATED FROM
Théorie Complette de la Conſtruction et de la
Manœuvre des Vaiſſeaux,
OF THE CELEBRATED
LEONARD EULER,
By *HENRY WATSON*, Eſq;

LONDON:
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MDCCLXXVI.

G R III
BRITISH MUSEUM

THE AUTHOR'S DEDICATION.

TO

HIS IMPERIAL HIGHNESS

PAUL PETROVITZ

GRAND DUKE OF ALL THE RUSSIAS

REIGNING DUKE OF SLESWICK HOLSTEIN

GREAT ADMIRAL OF RUSSIA.

&c. &c. &c.

MY LORD

THIS ſmall Work, which I take the Liberty of dedicating to your Imperial Highneſs with the moſt profound Reſpect, treats of a Subject which ſeems ſufficient to apologiſe for my Preſumption.

The Science of whatever relates to Navigation is without doubt one of the moſt ſublime and moſt uſeful Em-

Employments of the Human Mind; nevertheleſs, it has been hitherto almoſt entirely neglected; and altho' forty Years have elapſed ſince Mathematicians have laboured with ſome Succeſs, yet their Diſcoveries are ſo much enveloped in profound Calculations, that Mariners have ſcarce been able to derive any Benefit from them.

I flatter myſelf with having found the Means of placing all their Reſearches within the Reach of thoſe who apply themſelves to Marine Affairs; and it muſt be allowed that an exact Knowledge of the true Principles and Reaſons upon which the Goodneſs of the Conſtruction of Ships is founded, muſt conduce to render the Practice perfect, and to remedy all the Defects to which it is ſtill liable.

The

THE AUTHOR'S DEDICATION.

The Utility which may reſult from this Work, has emboldened me to lay it at the Feet of your Imperial Highneſs, and I dare to hope that you will be pleaſed to accept the Homage of it.

I am with the moſt profound Reſpect,

MY LORD

Your Imperial Highneſs's

moſt humble and

moſt obedient Servant

Peterſbourg
Nov. 30, 1773.

L. EULER.

T O

THE RIGHT HONOURABLE

JOHN EARL OF SANDWICH

VISCOUNT HINCHINGBROOK

FIRST LORD COMMISSIONER OF THE ADMIRALTY

ONE OF HIS MAJESTY'S MOST HONOURABLE
PRIVY COUNCIL,

&c. &c.

MY LORD

THE great Variety of important Knowledge contained in *Euler*'s *Theory of the Conſtruction and Management of Ships*, and a Wiſh to extend the Benefit of his Labours to ſuch of my Countrymen as are not converſant in the Technical Language of the Original, have been my ſole Motives for attempting an Engliſh Tranſlation of that celebrated Work; I ſhall be happy if the Succeſs of the learned Profeſſor in treating the natural Difficulties of the Subject may excite the Navigators and Artiſts of this Kingdom to render his Theory more perfect, and to become as eminently Skilled in the ſcientific as they now confeſſedly are in all the practical Branches of their Profeſſion.

With the ſame patriotic View I preſume to addreſs the following Sheets to your Lordſhip, in full Perſuaſion that nothing can ſo effectually recommend them to public Attention as the Approbation and Patronage of the Firſt Commiſſioner of the Admiralty; by whoſe Abilities and Application the Naval Power of Great Britain appears, at this Critical Period, in the moſt reſpectable State and Condition.

I am proud of this Opportunity of manifeſting my Duty and Affection for my

King and Country, as well as the particular Eſteem and Reſpect with which I have the Honour to be,

MY LORD

Your Lordſhip's

moſt obedient

and moſt faithful

humble Servant

Hanover-Street
December 1, 1776.

Henry Watſon.

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BOOK III.

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A
COMPLEAT THEORY
OF THE
CONSTRUCTION AND PROPERTIES
OF
VESSELS.

BOOK I.

In which Veſſels are conſidered in Equilibrium and at Reſt.

CHAP. I.

Of Veſſels in general.

§ I.

CHAP. I.

WHATEVER Differences there may be in the Conſtruction of the Veſſels which we uſe in Navigation, we always find this general Property, that each Veſſel is compoſed of two Parts perfectly equal, and joined in the Middle lengthwiſe; conſequently there is always a Section which divides the Veſſel into two ſimilar and equal Parts: This Section, made from Head to Stern through the Middle of the Veſſel, will be called the *Diametrical Section*; and that Half, which is ſituated on the Right of this Section, will be the *Starboard*, and that on the Left, the *Larboard*; ſuppoſing them to be viewed in Directions from the Stern towards the Head of the Veſſel.

§ 2.

Since theſe two Parts are not only alike between themſelves, but as care is alſo taken to load them equally, the Center of Gravity of the whole Veſſel muſt neceſſarily fall in the diametrical Section; and it is of the laſt Importance to know exactly the Place of this Point; which we ſhall hereafter call by the Name of the *Center of Gravity*.

§ 3.

When the Veſſel is placed in its true Situation, and in Equilibrium, the diametrical Section muſt always be vertical or perpendicular to the Horizon; beſides, there will ſtill be a Line parallel to the Horizon, which paſſing through the Center of Gravity, will be directed from the Stern to the Head of the Veſſel; and we ſhall name this Line the *Principal Axe of the Veſſel with reſpect to its Length*; and drawing a vertical Line through the Center of Gravity, it will be named the *Vertical Axe of the Veſſel*; finally, a third Line perpendicular to theſe two Axes, and paſſing through the Point of Interſection, will be called the *Axe of the Veſſel with reſpect to its Breadth.* It is to theſe three Axes, which interſect each other perpendicularly in the Center of Gravity, that we muſt pay our principal Attention, when it is required to determine all the Motions of which a Veſſel is ſuſceptible.

§ 4.

It is known that the Center of Gravity is the Point where the Weight of the whole Veſſel is collected, or through which paſſes the mean Direction of all the Forces of Gravity, wherewith all the Parts of the Veſſel are animated; therefore, having the whole Weight of the Veſſel, we know that it is puſhed by a Force equal to that Weight, whoſe Direction is found preciſely in the vertical Axe of the Veſſel, and tends towards the Center of the Earth.

§ 5.

Beſides the three Axes of which we have ſpoken, it is proper alſo to conſider three principal Sections in each Veſſel, the firſt of which is determined by the Principal Axe with reſpect to the Length, and by the Vertical Axe;

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from whence it is plain, that this Section is the ſame as that which we have already named diametrical: The ſecond principal Section is determined by the Axe with reſpect to the Breadth, and by the Vertical Axe; it is therefore alſo truly vertical like the preceding one; but ſince it is made breadthwiſe, it is called the Tranſverſe Section. Finally, the third principal Section made by the two horizontal Axes with reſpect to the Length and Breadth, will be alſo horizontal, and every where parallel to the Level of the Sea, when the Veſſel is in Equilibrium.

§ 6.

The Conſideration of theſe three principal Sections is ſo much the more important, as it comprehends a ſufficient Knowledge of the Figures of all Veſſels: For although theſe three Sections do not really determine the Figure of the Veſſel, and may belong to an infinity of different Kinds, yet all theſe Differences cannot exceed certain Limits ſufficiently confined; ſo that whatever Idea we might form of the Figure of the Veſſel, it cannot deviate conſiderably from the Truth.

§ 7.

Plate I. Fig. 1.

In order to explain this better, let Fig. 1. repreſent to us the Figure of any Veſſel whatever; of which let I A C B K be the diametrical Section,

tion, and the Point G its Center of Gravity; through which let be drawn in the ſame vertical Plane, the Axe A G B with reſpect to the Length, and the vertical Axe D G C; to which let be drawn perpendicular the third Axe E G F horizontally acroſs the Veſſel, ſo that theſe three principal Axes will be AGD, DGC, and E G F. Then, beſides the diametrical and vertical Section A C B, let A E B F be the Figure of the horizontal Section of the Veſſel, through the Center of Gravity D, and E C F the Tranſverſe Section; it is evident, that as ſoon as theſe three Figures are determined, the Form of the entire Veſſel can then have but little Variation: however, in order to ſhew it exactly, we have only to deſcribe ſome other vertical Sections parallel to the tranſverſe one, both towards the Head and Stern, and the greater this Number is, the more ſhall we approach to the true Figure of the Veſſel. Theſe Sections, parallel to the tranſverſe ones, bear the Name of *Gabaris*.

CHAP. II.

CHAP. II.

Upon the Floating of a Veſſel, or its State of Equilibrium in general.

§ 8.

Fig. 2. LET us at preſent ſuppoſe our Veſſel ſwimming upon the Water in Equilibrium, and in its proper Situation, where the Point G denotes the Center of Gravity, and the right Line D G C, the vertical Axe of the Veſſel. Here it is neceſſary at firſt to obſerve a new horizontal Section of the Veſſel, upon a Level with the Water, which is repreſented by the horizontal Line M H N, by which the Veſſel is divided into two Parts, the one above and the other within the Water, which we call the immerſed Part, and ſometimes alſo the *Hollow* of the Veſſel, or the Body.

§ 9.

In order to judge of this State of Equilibrium, in which we ſuppoſe the Veſſel to be, it is neceſſary to take into Conſideration all the Forces which act upon the Veſſel: And firſt, the proper Weight of the whole Veſſel preſents itſelf, by which it is preſſed down vertically in a Direction of the vertical Axe G C paſſing through the Center of Gravity of the

Veſſel. This Force muſt therefore be counterbalanced by all the Efforts which the Water exerts upon the Surface of the immerſed Part; and by conſequence it would be neceſſary to determine the Preſſure that each Particle of the immerſed Surface ſuſtains from the Water, which requires Reſearches very embarraſſing, and a long Series of Calculations: But the following Conſideration will eaſily lead us to the deſired End.

§ 10.

As the Veſſel occupies in the Water, by its immerſed Part, the Cavity MCN; let us compare this Caſe with another, where the ſame Cavity M C N was filled with Water, or rather with a Maſs of Ice, of the ſame Figure and Size (without conſidering the Difference between the ſpecific Gravities of Water and Ice); it is at firſt evident, that this Maſs of Water or Ice will be in a perfect Equilibrium with the Water which ſurrounds it; and it is alſo plain, that this Maſs ſuſtains from the Part of the ſurrounding Water, the ſame Efforts which our Veſſel ſuffers from it. From whence we ſee, that theſe Efforts of the Water balance the Weight of the Maſs of Water or of Ice which we have juſt ſubſtituted in the Place of the Veſſel: Therefore, ſince theſe ſame Efforts ſuſtain alſo the Weight of the whole Veſſel; it follows, That this Weight is preciſely equal to the Weight of the Maſs of Water which fills the ſame Cavity M C N; or rather, whoſe Volume is equal to the Volume of the immerſed Part of the Veſſel.

§ 11.

Here therefore is the firſt great Principle, upon which is founded the Theory of the Floating of Bodies that ſwim upon the Water: It is, That the immerſed Part muſt always be equal in Volume to a Maſs of Water, which would have the ſame Weight as that of the Veſſel: And it is from this Principle that we determine the true Weight of a Veſſel by meaſuring the Volume of its immerſed Part in the Water; for then, by reckoning about 64lb. Avoirdupoiſe for each Cubic Foot, we ſhall find the Weight of the Veſſel expreſſed in Pounds: But, in our Reſearches, it will be more convenient to expreſs the Weight of each Veſſel by the Weight of a Volume of Water equal to the immerſed Part.

§ 12.

However, this Principle, only, is not ſufficient to determine the State of the Equilibrium of a Veſſel: another muſt be ſtill joined to it; and which we ſhall find with the ſame Facility. We have only to conſider (in Fig. 3.) the Center of

Fig. 3.

of Gravity of the Maſs of Water MCN; which ſuppoſe at the Point O: then we conceive eaſily that all the Efforts of the ſurrounding Water are in Equilibrium with a Force equal to the Weight of the Maſs of Water M C N, which ſhould act in the perpendicular Direction O C downwards: Therefore, in order that our Veſſel may be in Equilibrium with the ſame Efforts, it is neceſſary that the Center of Gravity G of the Veſſel be in the ſame vertical Line H C, in which the Point O is ſituated: For that Purpoſe we have only to mark within the Veſſel the Point O, where the Center of Gravity of the immerſed Part would be, if it was compoſed of an homogeneous Matter.

§ 13.

In order to avoid long Circumlocutions, we ſhall name this Point O, ſimply, the *Center of the immerſed Part*, or rather, quite ſhort, the *Center of the Hollow*. Now the State of the Equilibrium of a Veſſel will be determined from theſe two Principles: 1ſt, That the immerſed Part muſt be equal in Volume to a Maſs of Water, whoſe Weight would be equal to that of the Veſſel; and 2dly, That the Center of Gravity G, and the Center O of the Hollow, fall in the ſame vertical Line H C, which is the vertical Axe of the Veſſel. With reſpect to this Point O, it is evident that it muſt always fall below the Water Line M N, and if the immerſed Part ſhould preſerve, in deſcending, every where the ſame Surface, or that it had either a priſmatic or cylindrical Figure, then the Point O would fall in the Middle of the Depth H C: But if the Extent diminiſhed uniformly, from the upper to the lower Part, and that it at laſt terminated in a right Line drawn through C equal and parallel to M N, then the Elevation of the Point O, or rather the Interval C O, would be Two-Thirds of the whole Depth O H; and if the ſame immerſed Part ſhould terminate in a Point C as a Pyramid reverſed, then the Interval C O would be three Quarters of the Depth H C: But with reſpect to the Center of Gravity G, it may fall either above or below the Water Line M N, according as the Lading ſhall be diſtributed throughout the Body of the Veſſel. Thus in Ships of War, where the Guns conſtitute a conſiderable Part of the Weight, ſince they are placed above the Water, the Center of Gravity G will be ſituated above its Surface.

CHAP. III.

Upon the Efforts of the Water to bend the Veſſel.

§ 14.

WHEN we ſay that the Preſſure of the Water upon the immerſed Part of a Veſſel counterbalances its Weight, we ſuppoſe that the different Parts of a Veſſel are ſo cloſely connected together, that the Forces which act upon its Surface are not capable of producing any Change; for we eaſily conceive, if the Connection of the Parts was not ſufficiently ſtrong, the Veſſel would run the Riſque either of being broken in Pieces, or of ſuffering ſome alteration in its Figure.

§ 15.

Fig. 4.

The Veſſel is in a Situation ſimilar to that of a Rod AB, which, being acted upon by the Forces A*a*, C*c*, D*d*, B*b*, may be maintained in Equilibrium, provided that it has a ſufficient degree of Stiffneſs; but as ſoon as it begins to give way, we ſee that it muſt bend in a convex Manner, ſince its Middle would obey the Forces C*c* and D*d*, whilſt its Extremities would be actually drawn downwards by the Forces A*a* and B*b*.

§ 16. The

CHAP. III.

§ 16.

The Veſſel is generally found in ſuch a Situation; and ſince ſimilar Efforts continually act whilſt the Veſſel is immerſed in the Water, it happens but too often that the Keel experiences the bad effect of a Strain: It is therefore very important to inquire into the true Cauſe of this Accident.

§ 17.

For this Purpoſe, let us conceive the Veſſel divided into two Parts by a tranſverſe Section through the vertical Axe of the Veſſel, in which Fig. 5. both the Center of Gravity G of the whole Veſſel, and that of the immerſed Part O, are ſituated; ſo that one of them will repreſent the Head Part, and the other that of the Stern; each of which we ſhall conſider ſeparately. Let g be therefore, the Center of Gravity, of the entire Weight of the firſt, and o that of the immerſed Part correſponding. In the ſame manner, let γ be the Center of Gravity of the whole Stern Part, and ω that of its immediate Portion.

§ 18.

Now it is plain, that the Head will be acted upon by the two Forces gm and on, of which the firſt will preſs it down, and the latter puſh it up. In the ſame Manner, the Stern will be preſſed down by the Force $\gamma\mu$, and puſhed up by the Force $\omega\nu$: But theſe four Forces will maintain themſelves in Equilibrium, as well as the total Forces re-united in the Points G and O, which are equivalent to them; but whilſt neither the Forces before, nor thoſe behind, fall in the ſame Direction, the Veſſel will evidently ſuſtain Efforts tending to bend the Keel upwards if the two Points $o\,\omega$ are nearer the Middle than the two other Forces $g\,m$ and $\gamma\mu$; A contrary Effect would happen, if the Points o and ω were more diſtant from the Middle than the Points g and γ.

§ 19.

But the firſt of theſe two Caſes uſually takes place almoſt in all Veſſels, ſince their Hollow has a greater Breadth towards the Middle, and becomes more and more narrow towards the Extremities; whilſt the Weight of the Veſſel is in Proportion, much more conſiderable towards the Extremities, than at the Middle. From whence we ſee, that the greater this Difference becomes, the more alſo will the Veſſel be ſubject to the Forces which tend to bend its Keel upwards: It is therefore from thence that we muſt judge how much Strength it is neceſſary to give to this Part of the Veſſel, in order to avoid ſuch a Conſequence.

§ 20. If

§ 20.

If other Circumſtances would permit either to load the Veſſel more in the Middle, or to give to the Part immerſed a greater Hollow towards the Head and Stern, ſuch an Effect would no longer be feared: But the Deſtination of moſt Veſſels is entirely oppoſite to ſuch an Arrangement; by which Means we are obliged to ſtrengthen the Keel as much as may be neceſſary in order to avoid ſuch a Diſaſter.

CHAP. IV.

Upon the three different Species of Equilibrium.

§ 21.

AFTER having eſtabliſhed the Requiſites of the Equilibrium of a floating Body, let us ſee what muſt happen when the Veſſel is ſomething diverted from that State. We ſhall ſuppoſe, at firſt, that the Veſſel ſuffers an exceeding ſmall Inclination from its natural Situation, in order to draw the Concluſions which are neceſſary to form a proper Judgment of its State of Equilibrium; for with reſpect to great Inclinations, which might become dangerous to a Veſſel, they require Reſearches entirely different.

§ 22. As

CHAP. IV.

§ 22.

As ſoon as a Veſſel becomes ever ſo little inclined or diſplaced from its State of Equilibrium, it is evident that three Conſequences may happen: 1ſt, Either the Veſſel remains in this inclined State, in which Caſe we ſay the Equilibrium is inſenſible: Or, 2dly, It re-eſtabliſhes itſelf in its preceding Situation, when its Equilibrium will be permanent; or rather, it will be endowed with a Stability which may be great or little, according to Circumſtances: Or, 3dly, The Veſſel, after this Inclination, will be entirely overturned; an Equilibrium of this Kind is like to that of a Needle, which put upon its Point, falls the Moment it has received the leaſt Motion: This Equilibrium is called unſtable, or ready to fall. We ſee evidently, that neither this laſt Caſe, nor the firſt, can have place in Veſſels; and with reſpect to the ſecond Caſe, a ſufficient Stability is abſolutely requiſite.

§ 23.

Plate II. Fig. 6.

In order to undertake the Reſearches neceſſary for this Subject, let us conſider any Veſſel whatever in its State of Equilibrium, of which let G be the Center of Gravity, and O that of the Part immerſed: Let the right Line MN repreſent the Section made at the Level of the

Water, the Line G O being vertical. Suppoſe now the Veſſel to be inclined in ſuch a Manner, that the Line *m n* becomes horizontal, or at the Surface of the Water, and that *m* D *n* be at preſent the Part immerſed; by which Means the Veſſel near M will be lowered by the Part M *m*, whilſt the other Side near N, will be raiſed by the Part N *n*. We muſt conceive beſides, that the new-immerſed Part, *n* D *m*, has the ſame Capacity as that of N D M in the State of Equilibrium, ſeeing that it is the firſt Condition requiſite for that State.

§ 24.

By this Change the Center of Gravity G of the Veſſel will ſtill remain the ſame; but with reſpect to that of the *Hollow*, which is increaſed on the Side M, and diminiſhed on the Side N, its Center of Gravity muſt neceſſarily be removed towards the Side M. Suppoſe *o* to be that Point; then, ſince the Line *mn* is at preſent horizontal, let fall the Perpendiculars G γ, *o* ω, then it is at firſt evident, that if the two Points γ and ω were united, the two Centers G and *o* would ſtill fall in the ſame vertical Line, and therefore the Veſſel would likewiſe be ſtill in Equilibrium; which is the firſt Caſe beforementioned of an inſenſible Equilibrium. From thence it is alſo plain, that this Caſe cannot happen but when the Point G is conſiderably elevated above the Point O.

§ 25. Let

§ 25.

Let us in the ſecond Place, conſider the Caſe repreſented in the Figure, where the Point γ is leſs diſtant from the vertical Line G O than the Point ω. Since the Veſſel is urged down according to the Direction G γ, and preſſed up by an equal Force in the Direction $o\,\omega$, it is evident that the Part towards M muſt by this means be raiſed, and of conſequence become leſs immerſed; or rather the Veſſel will return into its preceding ſtate of Equilibrium: This is therefore our ſecond Caſe of a permanent Equilibrium, whoſe Stability will be ſo much the greater the more diſtant the two Points γ and ω are from each other; and we have only to conſider the Figure, in order to aſſure ourſelves that the lower the Center of Gravity G is ſituated, the greater will be the Stability.

§ 26.

The third Caſe of an unſtable Equilibrium will happen when the Point γ is ſituated nearer the Extremity *m* than the Point ω; for then the two Forces will occaſion the Part M *m* to be more immerſed, and the Veſſel muſt by that means be entirely overturned. This is to be feared in Proportion as the Center of Gravity G is more elevated above the bottom of the Veſſel: But we ſhall very ſoon ſee, that beſides the two Centers G and O, the Figure and Extent of the Section of the Veſſel upon a Level with the

Water,

Water, enters here principally into conſideration.

§ 27.

In the Figure we have repreſented the Inclination as made from the Stern N towards the Head M, or round the tranſverſe Axe of the Veſſel: but it is evident that the ſame repreſentation may be applied when the Veſſel is inclined from one ſide towards the other, or round the principal Axe; from whence we eaſily comprehend, that in order to form proper Concluſions regarding the ſtate of Equilibrium in a Veſſel, it is neceſſary to make reſearches reſpecting both the Axes; for the caſe might eaſily happen, that a Veſſel had ſufficient Stability with reſpect to one of theſe two Axes, whilſt its Equilibrium with reſpect to the other might be *indifferent*, or even *unſtable.* It is likewiſe as certain, that when a Veſſel ſhall have a ſufficient degree of Stability with reſpect to theſe two principal Axes, it will alſo have ſufficient with reſpect to all the other intermediate Axes round which the Veſſel might receive any Inclination.

CHAP. V.

Upon the Method of reducing the Stability to a determined Meaſure.

§ 28.

FROM what has been premiſed, we eaſily conceive how the Stability of one Veſſel may be greater or leſs than that of another: But to form a true and preciſe idea of it, the beſt method is to ſee what Force we muſt apply to a Veſſel, in order to maintain it in a given degree of Inclination. We ſee plainly, that the conſideration here is not of one abſolute Force, becauſe very different Forces might produce the ſame Effect, being applied to different Points; and it is evident, that we muſt underſtand here the Momentum of the Forces with reſpect to the Axe round which the Inclination is made.

§ 29.

But the Axe of Inclination is always a Horizontal Line, paſſing through the Center of Gravity of the whole Veſſel; ſince we know from Mechanics, that a Force, whoſe Direction paſſes through the Center of Gravity of any Body, does not impart to it the leaſt angular Motion whatever, but is entirely employed to produce a progreſſive Motion. Therefore, in order to give a Veſſel an Inclination round a Horizontal Axe, paſſing through its Center of

Gravity,

Gravity, or to maintain the Veſſel in ſuch a ſtate of Inclination, it is neceſſary that the Force which we employ there, ſhould produce a Momentum with reſpect to the ſaid Axe; and we know that ſuch a Momentum is found by multiplying the Force by its Diſtance from the Axe of Inclination; from whence it is evident that the greater this Diſtance is, the leſs Force will be required to produce the ſame Effect.

§ 30.

In order to ſet this Matter in a clearer Point of View, ſuppoſe the Point G to be the Center of Gravity of a Veſſel, and that the Line A B repreſents the Section made on a Level with the Water, when the Veſſel is in Equilibrium. Alſo ſuppoſe, that by any Cauſe whatever the Veſſel has been inclined in ſuch a Manner that the Line $a\ b$ is become even with the Surface of the Water, and that the Inclination is repreſented by the ſmall Angle $A\ I\ a = B\ I\ b$, which we ſhall put $= i$, the Part immerſed will then be deſcribed by the Portion $a\ L\ b$: Let alſo the perpendicular Line H G L repreſent a Maſt fixed in the Veſſel, and ſuppoſe a Force H K capable of maintaining the Veſſel in that inclined Situation, is applied to it above the Point G at H, which Force we ſhall call K. Then ſince the Interval G H repreſents the Diſtance of that Force from the Point G, or rather from the Axe of Inclination, the Momentum of the Force which

Fig. 7.

which we require will be expreſſed by the Product K × G H. This Momentum muſt therefore be equal to the Efforts which the Veſſel exerts in order to re-eſtabliſh itſelf in its State of Equilibrium.

§ 31.

It is plain that this Momentum of Force muſt depend not only upon all the Circumſtances of the Veſſel, and of the Axe round which the Inclination is made, but alſo principally on the degree of Inclination itſelf, which we deſcribe by the Angle A I $a = i$; and ſince we ſuppoſe this Angle i extremely ſmall, it is evident that the Momentum in queſtion muſt be proportional to the Sine of that Angle, ſeeing the more that Angle increaſes, the greater alſo muſt be the Force of the Veſſel, in order to re-eſtabliſh itſelf in Equilibrium; conſequently the Momentum of Force requiſite to maintain the Veſſel in its inclined State, will always have this Form S. t. Sin. i; where S denotes a certain abſolute Force, t a certain Line, and Sin. i, the Sine of the Angle i, the Radius being ſuppoſed Unity. Fig. 7.

§ 32.

But when we ſpeak of the Stability of a Veſſel with reſpect to a certain horizontal Axe paſſing through its Center of Gravity G, we form to ourſelves an Idea that no ways depends on the

Quantity of the Inclination, ſeeing that the ſame Idea muſt belong as well to the State of Equilibrium itſelf, as to all the poſſible Inclinations: Therefore, in order to fix that Idea, we have only to omit in the given Expreſſion the Factor Sin. *i*; ſo that the Product S. *t* will be employed to expreſs what we mean by the Term *Stability*; which will therefore always be the Product of a certain Force, or rather of a Weight S, multiplied by a certain Line *t*. Fig. 7.

§ 33.

This Expreſſion will therefore be very proper to give us a juſt and clear Idea of the Stability of Veſſels, and it will afford the Means of comparing together exactly the Degrees of Stability which may belong to all Veſſels, ſo as to pronounce that the Stability of ſuch a Veſſel is two, or three, or ſo many Times greater or leſſer than that of another, without regarding in any Manner the Quantity of the Inclination itſelf which the Veſſel ſuffered.

§ 34.

Having therefore fixed this Idea of the Stability of any Veſſel whatever with reſpect to any Horizontal Axe, an Idea which is comprized in the Expreſſion S. *t*, nothing is more eaſy than to determine for any Inclination *i* whatever, the Momentum of Force requiſite to maintain the Veſſel in that inclined Situation, or rather that whereby the Veſſel endeavours to re-eſtabliſh itſelf in Equilibrium; for we have only to multiply the Formula S. *t*. by the Sine of the Angle *i*, and then the Product S. *t*. Sin. *i* will expreſs the Momentum which we require. Thus for example, if the Inclination was one Degree, ſince the Sine of this Angle is nearly $\frac{1}{57}$ Part of the whole Radius or Unity, the Momentum of the Force ſought will then be $\frac{1}{57}$. S. *t*.

§ 35.

Afterwards, knowing this Value of the Stability S. *t*, it will be eaſy to determine the Inclination by which any Momentum of Force whatever relative to the ſame Axe will be capable of changing the Situation of the Veſſel. For let this Force H K be = K, its Diſtance G H from the Axe of Inclination = *k*, then the Momentum of that Force will be = K. *k*, but it is requiſite that K. *k* = S. *t*. Sin. *i*; from whence we have Sin. $i = \frac{K.\,k}{S.\,t}$; by which Means the Momentum of the propoſed Force K. *k* divided by the Stability, will furniſh us with the Sine of the Inclination ſought, and therefore the Inclination *i* itſelf. Fig. 7.

§ 36. Since

§ 36.

Since we do not here conſider but very ſmall Inclinations, and becauſe the Security of Navigation abſolutely requires that Veſſels be never expoſed to Inclinations that are too great; it is neceſſary that the Stability S. *t* be always ſeveral Times greater than the greateſt Momentum of the Forces K. *k* to which Veſſels can ever be expoſed. Here is therefore a Precept of the laſt Importance in the Conſtruction of Veſſels; that it is always neceſſary to procure for a Veſſel ſuch a Degree of Stability as may be many Times greater than the greateſt Efforts which the Veſſel can ever be expoſed to. Thus, if we ſhould require that the Inclination ſhall never ſurpaſs ten Degrees, whoſe Sine is about $\frac{1}{6}$, it is neceſſary that the Stability be at leaſt ſix Times greater than the ſaid Efforts to which the Veſſel might be ſubject.

§ 37.

After theſe Expoſitions upon the true Idea of Stability, there remains only to ſearch for all Veſſels, the true Value of our ſuppoſed Formula S. *t*. For this purpoſe it is neceſſary to examine well all the Circumſtances that may contribute to augment or diminiſh the Stability of Veſſels, which we ſhall endeavour to explain in as diſtinct a Manner as poſſible; notwithſtanding that this Reſearch commonly requires

Reaſonings of a very complicated Nature. This ſhall be the Subject of the following Chapter.

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CHAP. VI.

Upon the Determination of the Stability of Veſſels.

§ 38.

ALL the Efforts by which an inclined Veſſel endeavours to recover its State of Equilibrium, proceed entirely from all the Elementary Preſſures which the immerſed Part experiences from the ſurrounding Water; the Gravity of the Veſſel itſelf not contributing any thing towards it; ſince the mean Direction paſſes through the Center of Gravity, and therefore does not furniſh any Momentum for its Re-eſtabliſhment. But we have ſeen that all the Elementary Preſſures perfectly counterbalance the Weight of a Maſs of Water which would occupy the Volume of the immerſed Part of the Veſſel; and therefore we have only to regard the immerſed Part as a Maſs of Water, all the Particles of which are vertically puſhed upwards with as much Force as their Gravity preſſes them down.

§ 39.

Let us conceive therefore, that a Veſſel whoſe immerſed Part in a State of Equilibrium was

Fig. 7.

was the Portion ALB, to be inclined in ſuch a Manner that its immerſed Part is now become *a* L *b*; and let us conſider this Volume as filled with Water; we ſhall then have only to ſearch what Force each Particle of this Volume (being preſſed upwards with a Force equal to its Weight) would exert in order to re-eſtabliſh the Veſſel; and for this Purpoſe we have only to find the Momentum of each of theſe Forces with reſpect to the Axe round which the Inclination has been made. This Axe is always, as we have ſeen, a horizontal Line drawn through the Center of Gravity G of the Veſſel, which we conſider here as perpendicular to the Plane of the Figure which repreſents a Vertical Section through the Center of Gravity G.

§ 40.

In order to determine more readily all theſe Elementary Forces, let us decompoſe the immerſed Volume *a* L *b*, ſo that it may contain, firſt, the immerſed Part A L B in a State of Equilibrium; ſecondly, the Angular Volume A I *a*, which muſt be added to it; and thirdly, the Angular Volume B I *b*, which muſt be ſubtracted, in order to have the Immerſed Volume *a* L *b*. Let us ſearch at preſent what Force each of theſe three Portions furniſhes, in order to cauſe the Veſſel to turn round the before-mentioned Axe; for then we have only to add together the Forces, or rather the Momentums which reſult from the two firſt Portions, and deduct from their Sum the Momentum which ariſes from the latter.

§ 41.

Let us therefore begin by conſidering the Maſs of Water, which would fill the Space A L B, whoſe Center of Gravity is placed at the Point O, from whence we are permitted to conceive all this Maſs of Water as collected in the Point O, and puſhed vertically upwards with a Force equal to its proper Weight. But the Weight of this Maſs A L B, being equal to the Weight of the whole Veſſel, if we call this Weight M, we ſhall have at O a Force applied = M, which preſſes the Veſſel vertically upwards. But ſince the Line ab is at preſent horizontal, let Om be drawn perpendicular to it from the Point O, which will by conſequence be our vertical Line, or the Direction of our Force M: Therefore, in order to find its Momentum, let fall the perpendicular Gv upon the Line Om produced, and the Momentum of this Force will be $= M \times Gv$: But it is evident, ſince the Line OG was vertical in the State of Equilibrium or perpendicular to the Line AB, and the Line Ov is perpendicular to the Line ab; I ſay it is obvious, that the Angle GOv is equal to the Angle of Inclination AIa, which we ſhall call i: Therefore Fig. 7.

fore the Line Gv, divided by the Diſtance O G, will give the Sine of the Angle i, or rather we ſhall have G v = O G. Sin. i; conſequently the Momentum of this Force will be = M. O G. Sin. i.

§ 42.

Here is therefore the Momentum of the Force which reſults from the Portion of Water A L B, which we have juſt found = M. O G. Sin. i; where M repreſents the Weight of the whole Veſſel, and the Line O G the Elevation of the Center of Gravity G above the Center O of the Hollow, when in the State of Equilibrium. But as this Force preſſes upwards in the Direction O v, it will evidently tend to augment the Inclination, or to immerſe more the Part A L; ſo that this Force is contrary to the Re-eſtabliſhment of the Veſſel in Equilibrium. Therefore, if the two other Portions of Water A I a and B I b, which we have ſtill to conſider, ſhould not furniſh us with a Momentum of Force oppoſite to and greater than the Effects of the firſt, the Veſſel would not have any Stability, and the leaſt Inclination would overturn it entirely. This muſt be underſtood when the Point G is more elevated than the Point O, as it is repreſented in the Figure: For if the Point G ſhould fall below the Point O, then the Momentum of that Force would have a Tendency oppoſite to the re-eſtabliſhment of the Veſſel in Equilibrium.

brium. But this Caſe can ſeldom or ever happen in Veſſels.

§ 43.

Let us at preſent examine the angular Portion of Water A I *a*: Then, ſince it is neceſſary to take into conſideration all the Particles of Water ſituated in that Space, let us conceive in the Section of Water A B, which correſponds to the State of Equilibrium, any Particle P P whatever, extremely or almoſt infinitely ſmall; and let us conſtitute upon it the ſmall Column P P *p p* terminated by the Section of the Water *a b*, and which let be perpendicular to it. Now ſince we here ſuppoſe the Inclination *i* as infinitely ſmall, this Column might alſo be regarded as perpendicular to the Line A B, from whence the Heighth of this Column P *p* will be $=$ I P. Sin. *i*, and its Solidity $=$ P P. I P. Sin. *i*, which repreſents therefore the Volume of Water contained in that Column, whoſe Weight we muſt here conſider. For this Purpoſe, let us put the Volume of the immerſed Part of the Veſſel requiſite for the Equilibrium A L B $=$ V; then, ſince the Weight of ſuch a Volume of Water is preciſely equal to the Weight of the Veſſel M, we ſhall have only to make this Proportion: As the Volume V is to the Weight M, ſo is the Volume of the Column P P*pp* to its Weight, which will conſequently be $= \frac{M}{V}$. P P. I P. Sin. *i*. But, inſtead

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of this Expreſſion, we ſhall put, for an abridgement, the Letter T; ſo that

$$T = \frac{M}{V} . PP. \ IP. \ \text{Sin.}\ i.$$

§ 44.

Having now found the Weight of this ſmall Column of Water, P P *p p* which is = T; the Force which reſults from it is directed upwards, and perpendicular to the Line I *a*. In order to find the Momentum of it, let fall the perpendicular G *g* from the Point G upon the Line *a b*; then it is evident that this Momentum will be found by multiplying the Force T by the Diſtance *p g*; ſo that the Momentum in queſtion will be = T. *p g*; or rather, ſince $pg = Ip + Ig$, and the Line I *p* is nearly equal to the Line I P, becauſe the Angle *i* is infinitely ſmall; this Momentum of Force will be repreſented by $T.IP + T.Ig$; which Force plainly tends to diminiſh the Inclination, and to re-eſtabliſh the Equilibrium. Having determined in the ſame Manner the Momentums of all the Particles P P, which we may conceive in the Section of Water from I to A; the Sum of all theſe Expreſſions joined together will give the Momentum of Force which reſults from the Portion of Water contained in the angular Space A I *a*. Theſe Sums, according to the Fluxionary Method, may be therefore expreſſed by Fluent T. I P + Fluent T. I *g*, which denotes the total Momentum of the Forces, reſulting from that Portion A I *a*, in order to re-eſtabliſh the Veſſel in Equilibrium.

§ 45.

Let us in the ſame Manner conſider the Space B I *b* as filled with Water, and taking in the Section I B, any Particle whatever Q Q; let Q Q *q q* be the ſmall Column correſponding to it; and which we may regard as perpendicular, both to A I B and *a* I *b*; then the Solidity of this Column will be found as before $= QQ.\ IQ.\ \text{Sin.}\ i.$ from whence its Weight will be $= \frac{M}{V} \cdot QQ.\ IQ.\ \text{Sin.}\ i. = U$, by putting the Letter U, to denote this Weight. It is therefore to this Weight, that the Force of the Column will be equal; and ſince it acts vertically upwards, its Momentum, with reſpect to the Line of Inclination, will be $= U.\,qg$; but becauſe $qg = Iq - Ig$, and alſo $Iq = IQ.$ this Momentum will become equal to $U.\,IQ - U.\,Ig$; from whence the Sum of all theſe Forces joined together, will be expreſſed by Fluent $U.\,IQ -$ Fluent $U.\,Ig$: and as this Force is applied on the other Side the Point G, its Effect will tend to augment the Inclination; but ſince this Portion B I *b* muſt be deducted from the two preceding ones, its Effect muſt therefore be taken negatively; and therefore the Veſſel will experience, on account of this Portion, a Momentum of Force equal to that which we have juſt aſſigned, in order to re-eſtabliſh it in Equilibrium.

§ 46.

Let us now join together the Momentums of the Forces which reſult from the two angular Spaces A I *a* and B I *b*, and we ſhall obtain this Expreſſion; Flu. T. I P + Flu. T. I *g* + Flu. U. I Q.—Flu. U. I *g*: which is compoſed of four Parts; the ſecond and fourth of which we ſhall firſt examine, ſince both the one and the other contain the ſame Interval I *g*, which always remains conſtant whilſt the Points P and Q paſs over the Spaces I A and I B; and therefore theſe two Members may be repreſented by I *g*. Flu. T — I *g*. Flu. U; but ſince T repreſents the Weight of the Column P P *p p*, the Fluent of T will expreſs the Weight of the Maſs of Water contained in the Space A I *a*. In the ſame Manner the Fluent of U will expreſs the Weight of the Water contained in the Space B I *b*; conſequently, ſince the immerſed Part *a* L *b* in the inclined State, is equal to that which correſponds to the Equilibrium A L B, the two Portions beforementioned, Flu. T. and Flu. U. muſt be equal to each other; by which means, the ſecond and the fourth Members mutually deſtroy each other; and therefore the Momentum of the Force which reſults from the two angular Spaces A I *a* and B I *b* will be reduced to this Expreſſion, Flu. $T.IP$ + Flu. $U.IQ$, from whence we muſt ſubtract that which the Firſt Portion has furniſhed, or $M.OG.$ Sin. i, in order to have the Momentum of the whole Force, which tends to re-eſtabliſh the Equilibrium.

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§ 47.

Let us ſubſtitute at preſent, in place of the Letters T and U their Values, which are

$$T=\frac{M}{V}.PP.IP.\text{ Sin. }i\text{, and}$$

$$U=\frac{M}{V}.QQ.IQ.\text{ Sin. }i.$$

Then, ſince the Quantities $\frac{M}{V}$ and Sin. i. remain conſtant, whilſt the Points P and Q paſs over the Spaces I A and I B, the above Expreſſion or Flu. $T.IP$ + Flu. $U.IQ$ may be repreſented by

$$\frac{M}{V}\text{ Sin. }i.\text{ Flu. }PP.\overline{IP}^2+\frac{M}{V}\text{ Sin. }i.\text{ Flu. }QQ.\overline{IQ}^2,$$

and therefore the whole Momentum in order to re-eſtabliſh the Veſſel will be

$$\frac{M}{V}\text{ Sin. }i\times\overline{\text{Flu. }PP.\overline{IP}^2+\text{Flu. }QQ.\overline{IQ}^2}$$

—$M.OG.$ Sin. i, which is therefore the Value of the Expreſſion $S.t.$ Sin. i, eſtabliſhed in the preceding Chapter, therefore nothing more is neceſſary than to divide this Expreſſion

Fig. 7.

ſion by the Sin. *i*, in order to have the Stability of the Veſſel with reſpect to the ſuppoſed Axe; This Stability will therefore be repreſented by the Expreſſion

$$\frac{M}{V} \times \overline{\text{Flu. } P P. \overline{I P}^2 + \text{Flu. } Q Q. \overline{I Q}^2} - M.OG,$$

where the firſt Member depends principally on the Section A B of the Water, and on its Figure, which we ſhall explain more particularly in the following Chapter; where we ſhall treat on the Nature of this Formula,

$$\text{Flu. } P P. \overline{I P}^2 + \text{Flu. } Q Q. \overline{I Q}^2,$$

under the Name of the Momentum of the *Section of the Water*.

C H A P. VII.

Upon the Momentum of the Section of the Water.

§ 48.

Fig. 7. FROM the Manner in which we have juſt treated the Subject, the two Sections of the Water A B and *a b*, which belong to the Equilibrium, and to the inclined State of the Veſſel, are conſidered as ſimple Lines, and their Interſection I as a Point, which has been done in order not to confuſe the Figure, or trouble

the Imagination too much. But as theſe two Sections are in effect plane Surfaces, their Interſection will be a right Line, horizontal and parallel to the Axe round which the Inclination is made: We muſt therefore conceive this Line as perpendicular to the Plane of the Figure, and paſſing through the Point I; and now it is evident that the Terms Flu. PP. $\overline{\mathrm{I\,P}}^{2}$ and Flu. QQ. $\overline{\mathrm{I\,Q}}^{2}$ will expreſs the Sums of all the Particles contained in the Section of Water AB, each multiplied by the Square of its Diſtance from the ſaid Interſection.

§ 49.

Since the two Angular Spaces A I *a* and B I *b* are ſuppoſed equal to each other, it is eaſy to conclude, that the common Interſection muſt paſs through the Center of Gravity of the Section of the Veſſel made upon a Level with the Water, which we name ſimply the *Section of the Water*. Let this Section be therefore repreſented by the Figure ACBD, in which let the Line AB repreſent the Diameter paſſing from the Head A unto the Stern B. It is therefore in this Line that the Center of Gravity of this plane Surface is to be found, which ſuppoſe to be at the Point I, and let the right Line MN be drawn through this Point parallel to the Axe round which the Inclination Fig. 8.

CHAP. VII. clination is made; in order to find the Momentum of the Section of Water with reſpect to that Axe, we have only to conſider any Particle Z whatever, and multiply it by the Square of its Diſtance from the Axe M N, or rather by $\overline{ZX}^2$, and the Sum of all theſe Products, taken throughout the whole Figure A C B D, on both Sides the Axe M N, will give us the Momentum ſought, which we have denoted before by the Sum of theſe two Terms, Flu. $PP.\overline{IP}^2 +$ Flu. $QQ.\overline{IQ}^2$. We may therefore at preſent expreſs it more conciſely by the Term Flu. $Z.\overline{ZX}^2$; by this Means the Stability of the Veſſel with reſpect to the propoſed Axe will be $= \frac{M}{V}\cdot$ Flu. $Z.\ \overline{ZX}^2 - M.OG$, where M repreſents the Weight of the whole Veſſel, V the Volume of its immerſed Part, and O G the Elevation of its Center of Gravity G above the Center of the Hollow O.

§ 50.

It is evident the Veſſel may be ſo inclined, that the Line M N ſhall remain immoveable, whilſt the Portion M A N is plunged into the Water, and the other Part elevated above the Surface; and ſince this Line M N always paſſes through the Center of Gravity I of the Section of Water, this Point will be preciſely the Point of Support which Mr. *de la*

Croix had ſought heretofore with ſo much Care: And it is not contrary to the Manner in which we conſider the Subject, when we refer all the Inclinations to the horizontal Axes which paſs through the Center of Gravity G. We have not ſaid that the Center of Gravity G remains immoveable during the Inclination, as it is a known Property in Mechanics, that an Inclination round any Axe whatever may be always reduced to an equal Inclination made round the Center of Gravity, provided that we give a ſuitable Motion to this Center. But when Forces capable of producing ſuch an Inclination are required, we muſt always refer them to the Axe drawn through the Center of Gravity, and never to the Line M N, although it ſhould remain fixed during the Inclination of the Veſſel.

§ 51.

It will appear at firſt that this Reſearch is ſubject to almoſt inſurmountable Difficulties; ſince on the one Hand it is neceſſary to collect all the Products of $Z. \overline{ZX}^2$ into one Sum through the whole Extent of the Section of Water A B C D, and on the other, that we muſt repeat this Operation for each Axe ſeparately: But we ſhall find the Means of removing all theſe Difficulties with eaſe; for, with reſpect to the laſt, it will vaniſh the Moment we reflect that it is ſufficient to ſeek only two Momentums of this Section of the Water, the one with reſpect to its great Axe AB, and the other with regard to its leſſer Axe CD, becauſe having found theſe two Momentums, we are able thereby to determine the Momentum with reſpect to each intermediate and oblique Axe MN: this Determination is founded upon the Principles of Elementary Geometry, as will be ſeen in the following Article; firſt obſerving that the great Axe AB is always directed from the Head to the Stern, and the leſſer Axe CD perpendicular to it, ſeeing the Deſtination of every Veſſel requires that the Length AB ſhould ſurpaſs the Breadth CD conſiderably.

§ 52.

Let us ſuppoſe therefore that the two Momentums of the Water Section with reſpect to the two principal Axes AB and CD, are already determined; let the firſt be repreſented by [AB] and the latter by [CD], then, in order to find the Momentum with reſpect to any other Axe MN whatever, which we ſhall expreſs in like manner by [MN], let us put the Declination of this Axe to the firſt AB, or rather the Angle $AIM = \theta$, and we ſhall always have this Determination,

$$[MN] = [AB]\ \text{Coſ.}\ \theta^2 + [CD]\ \text{Sin.}\ \theta^2;$$

where the Coſ. θ^2 expreſſes the Square of the Coſine

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Coſine of the Angle θ, and Sin. θ^2, the Square of the Sine of the ſame Angle θ: from whence we ſee, that in the Caſe where the Angle $\theta = o$, the Axe M N falls upon A B, becauſe the Coſ. $\theta = o$, and the Sin. $\theta = 1$, we have then $[MN] = [CD]$ as the nature of the thing requires. It is not neceſſary to demonſtrate this truth, ſince it depends on pure Geometry; and the Demonſtration is eaſily obtained. By making uſe therefore of this Sign, the Stability of the Veſſel, with reſpect to the Axe M N, will be $= \frac{M}{V} . [MN] - M . OG$.

§ 53.

The whole is therefore reduced to find the Momentum of a propoſed Section of Water, with reſpect to theſe two principal Axes A B and C D, or the Values of the Expreſſions [A B] and [C D]: But as that would require an exact knowledge of the Figure of the whole Section, which we could ſeldom or ever arrive at, we ſhall confine the Application of this Reſearch to two principal Figures between which the true Figure of every Section of Water is at all times contained. For let the firſt be a right-angled Parallelogram *a a b b*, (ſee Fig. 9.) and the other a Rhombus, A C B D, (ſee Fig. 10.) whereof both the one and the other have the ſame principal Axes A B and C D, as the propoſed Section of Water; it is then evident, that the true

Plate III. Fig. 9 & 10.

true Section of Water will be always leſs than the firſt of theſe two Figures, and always greater than the ſecond; by this means, when the Momentums of theſe two Figures are determined, that of the true Section will always hold a certain Mean between theſe two Limits, nearer to the one or the other, according to the Figure of the Veſſel; and in each Caſe it will not be very difficult to diſcover the true Mean, or a near Approximation, which without doubt is ſufficient for Practice.

§ 54.

Let therefore in the firſt place the Rectangle Fig. 9. *a a b b* repreſent the Section of the Water which is to be conſidered, whereof A B is the greater, and C D the leſſer Axe, and which interſect each other at I, the Center of Gravity of this Figure: Then having calculated the Sums of of all the Elementary Products beforementioned, we find the Momentum of this Figure, with reſpect to its great Axe A B, or the Value of $[AB] = \frac{1}{12} . AB . \overline{CD}^3$: and with reſpect to that of C D, the Momentum will be found $[CD] = \frac{1}{12} . CD . \overline{AB}^3$; from whence it is evident, that the latter of theſe two Momentums is much greater than the former, and that in Proportion as the greater Axe ſurpaſſes the leſſer. Thus, for Example, if the great Axe A B was four times greater than the ſmall one, or A B=4 C D, the firſt Momentum will be to the ſecond as 1 to 16; and in general theſe two Momentums will be to each other in the inverſe Ratio of the Squares of their Axes, that is to ſay, we ſhall always have this Proportion,

$$[AB] : [CD] :: \overline{CD}^2 : \overline{AB}^2.$$

§ 55.

Let the Rhombus A B C D now repreſent the Section of the Water, whoſe two Axes alſo interſect each other in the Center of Gravity I of the Figure; then, after making the neceſſary Calculations, we find the Momentum, with reſpect to the great Axe A B, or the Value of $[AB] = \frac{1}{48} AB.\overline{CD}^3$, and the Momentum, with reſpect to the ſmall Axe, or $[CD] = \frac{1}{48} CD.\overline{AB}^2$; ſo that theſe two Values do not differ from thoſe of the preceding Caſe, but in the Numerical Coefficient, which in the firſt Caſe was $\frac{1}{12}$, and in this $\frac{1}{48}$, or one fourth of the former, which is worthy of Notice here; ſince the Area of this Figure is preciſely half that of the preceding one. From whence it already appears probable, that the Numerical Coefficients, for all other Figures whatever are in Proportion as the Squares of their Areas, whilſt the ſame Expreſſions $AB.\overline{CD}^3$ and $CD.\overline{AB}^3$ will enter equally into the Momentums of all Figures.

Fig. 10.

§ 56.

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Plate II. Fig. 8.

§ 56.

In order to examine this Rule, let us give to the Section of Water the Figure of an Ellipſe, (ſee Fig. 8.) whoſe great Axe is in like manner $=$A B, and the ſmall one $=$C D; then having made the neceſſary Calculations, according to Methods well known, in which the Quadrature of the Circle is found to enter; where, if we put $\pi = 3{,}14159265 =$ Circumference of a Circle, whoſe Diameter is Unity, we ſhall have the Momentum of this Ellipſe, with reſpect to its great Axe A B, or $[AB] = \frac{\pi}{64}. AB. \overline{CD}^3$, and the Momentum, with reſpect to the ſmall Axe, or $[CD] = \frac{\pi}{64}. CD. \overline{AB}^3$; the Numerical Coefficient is here $\frac{\pi}{64}$, whilſt in the Caſe of the Rectangle it is $\frac{1}{12}$; therefore theſe two Coefficients are as $\frac{\pi}{64} : \frac{1}{12}$, or as $\pi : \frac{16}{3}$. But the Area of the Ellipſe being $\frac{\pi}{4}. AB. CD$, it is to the Area of the Rectangle as $\pi : 4$; and therefore their Squares as $\pi\pi : 16$. It muſt be acknowledged, that theſe two Proportions are not perfectly equal; but the Difference is ſo ſmall, that in Practice we may boldly make uſe of the Rule mentioned in the preceding Article; ſeeing that the Nature of the thing itſelf is not ſuſceptible of a perfect Preciſion.

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§ 57.

Let us therefore make uſe of that Rule, and whatever Figure the Section may have, let us ſearch the Area of it, in order to compare it with the Area of the Rectangle *a a b b* formed by the ſame principal Axes of the propoſed Figure, and let us put $\frac{\text{Area}}{\text{AB.CD}} = \alpha$, ſo that we may in each Caſe regard this Fraction α as known, and then the Momentum of this Section of the Water, with reſpect to its great Axe A B, will be $[\text{AB}] = \frac{\alpha\alpha}{12}.\ \text{AB}.\ \overline{\text{CD}}^2$, and in the ſame manner the Momentum, with reſpect to its ſmall Axe $[\text{CD}] = \frac{\alpha\alpha}{12}.\ \text{CD}.\ \overline{\text{AB}}^2$.

Plate III. Fig. 9.

Thus are the great Difficulties which at firſt preſented themſelves happily ſurmounted, and that in a manner ſo eaſy, that we may uſe it in Practice almoſt without any trouble. From thence we are able to give the Momentum for any other Axe M N of the propoſed Section of Water; for putting the Angle A I M as before $= \theta$, we ſhall at firſt ſight have the Momentum ſought $[\text{MN}] = \frac{\alpha\alpha}{12}.\ \text{AB}.\ \text{CD}.$ $\overline{\overline{\text{CD}}^2.\ \text{Coſ.}\ \theta^2 + \overline{\text{AB}}^2.\ \text{Sin.}\ \theta^2}$.

Plate II. Fig. 8.

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CHAP. VIII.

Conſiderations upon the other Principles which enter into the Determination of the Stability.

§ 58.

HAVING removed all the Difficulties reſpecting the Momentum of the Section of the Water, which contains the principal Element of the Stability of Veſſels, let us conſider alſo all the other Principles which concur to eſtabliſh the true Value of the Stability of Veſſels, in order that we may be able to determine how much all the Circumſtances of a Veſſel are capable either of augmenting or diminiſhing the Stability. For this Purpoſe let us conſider our Expreſſion, which gives the Value of the Stability with reſpect to any Axe M N whatever, and which is $\frac{M}{V}.[MN]-M.OG$.

§ 59.

At firſt we perceive that the Weight of the Veſſel M is a Factor of this Expreſſion, ſo that as long as the other Elements remain the ſame, the Stability is always proportional to the Weight of the Veſſel; by this Means a Veſſel, whoſe Dimenſions are twice as great, ſince its Weight will then be eight times greater, will alſo have its Stability augmented eight times. This is abſolutely neceſſary, ſince great Veſſels are expoſed to the Action of much greater Efforts; and as theſe Efforts are in proportion to the Surfaces of Veſſels, or the Square of their ſimple Dimenſions, and their Diſtances from the Axe of Inclination in proportion to theſe ſame Dimenſions, it is evident that the Momentums of theſe Efforts will be proportional to the Cube of theſe Dimenſions, or to the Weight M of the whole Veſſel, allowance being made for the Inequality or Diſſimilarity in their Figures and Conſtruction.

§ 60.

Let us now conſider the Volume of the immerſed Part denoted by the Letter V, which might well be regarded as equivalent to the Weight M of the Veſſel, ſince a Maſs of Water, whoſe Volume is = V, would have the ſame Weight M: but this Volume V does not here enter into Conſideration, except as a geometrical Extent of three Dimenſions. For this Purpoſe let ACBD be the Section of the Water of a Veſſel, whoſe two Axes AB and CD are the ſame as heretofore, and interſect each other in the Center of Gravity I of that Section; alſo let AEB be the vertical and diametrical Section of the Veſſel from the Section of the Water to the Keel, this Figure will then repreſent to us the Body of the Veſſel, whoſe Volume we ſuppoſe = V; let alſo the Center of Gravity of this Volume be ſtill placed at O, and the Center of Fig. 11.

of Gravity of the Veſſel itſelf at G, in order to have the Interval O G, which enters likewiſe into the Stability, ſince it may very well happen that this Line G O does not paſs through the Point I. We ſhall therefore now have the Depth of the Body or the Draught of Water repreſented by the vertical right Line I E, on which the Volume V principally depends; ſo that we may always conſider it as a Product of the Water Section itſelf by a certain Part of the Depth I E.

§ 61.

With reſpect to the Area of the Water Section, we have already remarked, that it is always leſs than the Rectangle of its two Axes AB and CD, and greater than a Rhombus, whoſe Diagonals are A B and C D; let us put therefore as before, this Area $= \alpha .\ AB.CD$, where α is a Fraction leſs than 1, and greater than $\frac{1}{2}$. Now it is plain, if the Body ſhould preſerve the ſame Heighth even to the Keel, or that the Tranſverſe Sections were Rectangles, the Solidity would be,

$$V = \alpha .\ AB.\ CD.\ IE.$$

But if all the tranſverſe Sections were Triangles, whoſe Points terminated in the Keel, then the Area ACBD ought to be only multiplied by half the Depth I E, ſo that we ſhould have

$$V = \frac{1}{2}\alpha.\ AB.\ CD.\ IE.$$

We muſt ſtill remark, that if the whole Body was a Pyramid reverſed, and terminated at the Point E, we ought to multiply the Area A C B D only by one Third of the Depth I E: But this laſt Caſe being entirely excluded from Practice, we may conſider all the Bodies as contained between the two preceding Caſes; ſo that it is always neceſſary to multiply the Section of the Water by ſome Part of the Depth or β. I E, where β alſo denotes a Fraction contained between the Limits 1 and $\frac{1}{2}$, the true Value of which, or nearly ſo, it will not be very difficult to eſtimate for each Caſe. From thence we ſhall have the Volume of the Body $V = \alpha\beta$. A B. C D. I E, ſo that this Expreſſion is always a certain Part of the Solid formed by the three Dimenſions A B, C D and I E. But the Coefficient $\alpha\beta$ will be always leſs than 1, and greater than $\frac{1}{4}$, ſince both the one and the other of theſe two Letters α and β are contained between the Limits 1 and $\frac{1}{2}$.

§ 62.

The Interval O G ſtill remains to be conſidered, which is compoſed of two Parts, O F and

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and F G, of which O F is determined by the Figure of the Body only, whilſt the other F G depends upon the Weight of the whole Veſſel; and according to different Deſtinations, it may happen, that the Center of Gravity G will be more or leſs elevated above the Section of the Water, or that it may even fall below it; in which Caſe the Interval F G would become negative: But with reſpect to the Interval O F, let us ſee what Proportion it will have to the Depth I E, in the three Caſes before mentioned. In the firſt, where we have $\beta = 1$, and where the Body has in every Part the ſame Breadth, it is plain that we ſhall have $OF = \frac{1}{2} IE$; in the ſecond Caſe, where $\beta = \frac{1}{3}$ and all the tranſverſe Sections are Triangles, we ſhall have $OF = \frac{1}{3} IE$; and finally, in the third Caſe, where $\beta = \frac{1}{3}$, we ſhall have $OF = \frac{1}{4} IE$. From whence we conclude, that if we had $\beta = \frac{1}{n}$, we ſhould have $OF = \frac{1}{n+1} . IE$, and therefore having already found the juſt Value of of β, we might always ſuppoſe the Interval $OF = \frac{\beta}{\beta+1} . IE$, ſo that our laſt Element would become

$$OG = \frac{\beta}{\beta+1} . IE + GF.$$

§ 63.

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Having inveſtigated all the Elements which enter into the Expreſſion of the Stability, we obſerve, 1ſt, That the Quantity [M N] contains four Dimenſions, or that it is a Product of four Lines multiplied together; and 2dly, that the Volume V is a Quantity of three Dimenſions; from whence it appears that $\frac{[MN]}{V}$ will expreſs a certain right Line, which being put $= l$, the Stability will be $= M \times \overline{l - OG}$, from whence we perceive, that this Line l muſt always be greater than the Interval O G, and as it depends on the Axe M N round which the Inclination is made, it will become the leaſt poſſible when the Axe M N coincides with the great Axe A B. It is therefore abſolutely neceſſary, that this leaſt Value of l be ſtill greater than the Interval O G. Hence, if the Stability of Veſſels with reſpect to the great Axe A B be ſufficiently great to reſiſt all the Efforts, we may be certain that for all other Axes, the Stability will be more than ſufficient.

§ 64.

If Veſſels were in all reſpects perfectly ſimilar to each other, and their Weights in Proportion to the Cubes of their ſimple Dimenſions, ſo that the difference $l - OG$ was itſelf in Proportion to theſe ſimple Dimenſions, their Stability would be in Proportion to the fourth Power of the ſimple Dimenſions. Therefore ſince the Efforts to which Veſſels are expoſed are in Proportion to the Cube of their Dimenſions, great Veſſels would in Proportion have more Stability than ſmall ones; and moreover, being agitated by like Efforts, they would receive a leſs Inclination than the ſmall ones: From whence it ſeems, that with reſpect to great Veſſels, we might very well diminiſh their Stability, and make it proportional to the Cube of their Dimenſions. But it is neceſſary alſo to remark, that equal Inclinations might become fatal to great Veſſels, at the ſame Time that ſmall ones would be out of Danger; from whence we muſt conclude, that large Veſſels require in Proportion a greater Stability than ſmall ones. But we ſhall examine this Matter more particularly in the following Chapter.

CHAP. IX.

Upon the Means of giving to Veſſels a ſufficient Degree of Stability.

§ 65.

HAVING already remarked, that the Momentum of a Section of Water, with reſpect to its great Axe A B, is the leaſt,

and

and with reſpect to the leſſer Axe C D the greateſt; and even the former is to the latter as $\overline{CD}^2$ to $\overline{AB}^2$: it is therefore evident that the Stability of a Veſſel with reſpect to its great Axe A B is alſo the leaſt, and with reſpect to its leſſer Axe the greateſt, and that in a Proportion ſtill greater than $\overline{CD}^2$ to $\overline{AB}^2$. For ſince the Stability with reſpect to the Axe A B has been found $= M \times \overline{\frac{[AB]}{V} - OG}$, and that with reſpect to the Axe C D $= M \times \overline{\frac{[CD]}{V} - OG}$ it is evident that theſe two Expreſſions bear to each other a greater Proportion than their firſt Parts $\frac{[AB]}{V}$ and $\frac{[CD]}{V}$, ſince each is diminiſhed by the Quantity O G. But it is alſo neceſſary that the laſt Stability ſhould be greater than the firſt, ſince the ſame Efforts or Shocks, when they are made upon the Head or Stern, produce a greater Momentum than when made upon the Sides of the Veſſel. But their Proportion is, at moſt, in the Ratio of A B to C D. Therefore ſince the Stabilities themſelves preſerve a much greater Proportion, it is plain, that when a Veſſel has a ſufficient Stability with reſpect to its great Axe A B, there can be no doubt of its having a ſufficient one with reſpect to its leſſer Axe C D: For which

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reaſon we ſhall in this Chapter only conſider the Stability with reſpect to the great Axe A B, and examine by what Means this Stability may be increaſed and carried to a Degree requiſite for the Safety of the Veſſel.

§ 66.

The Momentum of the Section of Water with reſpect to its great Axe A B, has been found $[A\,B] = \frac{\alpha\,\alpha}{12}.\ A\,B.\ \overline{C\,D}^3$, where α denotes the Fraction which is found by dividing the Area of the Section of Water A B C D by the Rectangle A B. C D, and where it is neceſſary to remember that α is always comprized between the Limits of 1 and $\frac{1}{2}$. Now we may always conceive the immerſed Volume V. as the Product of the Area of the Section of Water which has been denoted by $\alpha.\ A\,B.\ C\,D$, multiplied by a certain Part of the Depth I E, which Part we have put $= \beta.\ I\,E$; it muſt be obſerved alſo that β is always a Fraction contained between 1 and $\frac{1}{2}$; ſince the Firſt would take place if all the tranverſe Sections were Rectangles, and the Second if they were Triangles whoſe Points terminated at the Keel: It is true that in theſe Limits we have not taken into conſideration the Obliquity with which the Body riſes towards the Head and Stern, from whence there reſults, without doubt, ſome Diminution; however it does not appear that the Value of β can ever become leſs than $\frac{1}{2}$: At all events we ſhall leave this Letter β indeterminate; from thence we have the Volume $V = \alpha\beta.\ AB.\ CD.\ IE$, and therefore the firſt Term of the Stability $\frac{[AB]}{V}$ will be $= \frac{\alpha}{12\beta} \cdot \frac{\overline{CD}^2}{IE}$; where the Length AB vaniſhes out of the Calculation, but it enters again into the Weight M.

§ 67.

Afterwards with reſpect to the Interval $OG = OF + FG$ we have ſhewn that the Part OF may always be ſuppoſed $= \frac{\beta}{1+\beta} . IE$; conſequently our Expreſſion for the Stability, with reſpect to the Axe AB, will take this form.

$$M \times \frac{\alpha}{12\beta} \cdot \frac{\overline{CD}^2}{IE} - \frac{\beta}{1+\beta} \cdot IE - FG.$$

From whence upon the firſt view we draw this abſolute neceſſary Condition, that the Expreſſion $\frac{\alpha}{12\,\beta}\cdot\frac{\overline{CD}^2}{IE}$ muſt always be greater than the Quantity $\frac{\beta}{1+\beta}\cdot IE + FG$. For if theſe two Quantities were equal, the Equilibrium of the Veſſel would be *inſenſible*; and if the Firſt was leſs than the Second, the Equilibrium would be *unſtable*, and the Veſſel would be overturned by the leaſt Agitation. In order to render this Condition more intelligible, let both Parts be multiplied by $\frac{12\,\beta}{\alpha}\cdot IE$, that it may take this form:

$$\overline{CD}^2 > \frac{12\,\beta\,\beta}{\alpha.\,\overline{\beta+1}}.\,\overline{IE}^2 + \frac{12\,\beta}{\alpha}.\,IE.\,FG$$

From whence we obſerve, that the Square of the Breadth $\overline{CD}^2$ muſt always be greater than the Value of the Expreſſion

$$\frac{12\,\beta\,\beta}{\alpha.\,\overline{\beta+1}}\cdot\overline{IE}^2 + \frac{12\,\beta}{\alpha}.\,IE.\,F.G.$$

§ 68.

In order that this important Condition reſpecting the Stability of Veſſels may be the better underſtood, let us put, for the ſake of Abbreviation, $\frac{12\,\beta\,\beta}{\alpha.\,\overline{\beta+1}} = m$, and $\frac{12\,\beta}{\alpha} = n$, then our Condition will be expreſſed in this manner

$$\overline{CD}^2 > m.\ \overline{IE}^2 + n.\ IE.\ FG.$$

Now ſince the two Letters α and β may vary from 1 to $\frac{1}{2}$, and perhaps the laſt β become even leſs than $\frac{1}{2}$, we ſhall exhibit the Values of the Letters m and n in the following Table, where we give to α ſucceſſively the Values of 1; 0, 9; 0, 8; 0, 7; 0, 6; and 0, 5; and to β thoſe of 1; 0, 9; 0, 8; 0, 7; 0, 6; 0, 5; and 0, 4; which Values of m and n, reſulting from each Combination of the Values of α and β, we ſhall expreſs in Decimal Fractions.

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IX.

β	Values of α.						
	1,0	0,9	0,8	0,7	0,6	0,5	
1,0	6,00	6,67	7,50	8,57	10,00	12,00	$= m$
	12,00	13,33	15,00	17,14	20,00	24,00	$= n$
0,9	5,12	5,69	6,40	7,31	8,53	10,24	$= m$
	10,80	12,00	13,50	15,43	18,00	21,60	$= n$
0,8	4,27	4,74	5,34	6,10	7,12	8,54	$= m$
	9,60	10,67	12,00	13,71	16,00	19,20	$= n$
0,7	3,46	3,84	4,33	4,94	5,77	6,92	$= m$
	8,40	9,33	10,50	12,00	14,00	16,80	$= n$
0,6	2,70	3,00	3,38	3,86	4,50	5,40	$= m$
	7,20	8,00	9,00	10,29	12,00	15,40	$= n$
0,5	2,00	2,22	2,50	2,86	3,33	4,00	$= m$
	6,00	6,67	7,50	8,57	10,00	12,00	$= n$
0,4	1,37	1,52	1,71	1,96	2,28	2,74	$= m$
	4,80	5,33	6,00	6,86	8,00	9,60	$= n$

§ 69.

There can be no doubt but all the poſſible Caſes are contained in this Table, yet as the extreme Values of the Letters α and β are excluded from Practice, the actual Caſes, which uſually have place in Veſſels, muſt be ſought for towards the middle of this Table; and as nothing more is here required than to aſſign certain Limits which the Square of the Breadth C D muſt exceed, and that ſufficiently conſiderable; a ſcrupulous Preciſion would here be very ill placed. For which reaſon we have only to take care that theſe Limits are not eſtabliſhed too ſmall; and for this purpoſe it ſeems that the Caſe where α and β are each $= 0, 8$ is very proper to be applied almoſt to every Veſſel uſed in Navigation. In this Caſe therefore we ſhall have $m = 5, 34$ and $n = 12, 00$, and our other Limit will be

$$\overline{CD}^2 > 5, 34. \overline{IE}^2 + 12. IE. FG.$$

However, in order to ſhew how a ſmall difference would influence the Value of $\overline{CD}^2$, we ſhall here add another Caſe, where $\alpha = 0, 7$ and $\beta = 0, 7$, which will give us this Limit

$$\overline{CD}^2 > 4, 94. \overline{IE}^2 + 12. IE. FG.$$

§ 70.

In the preſent Caſe all depends upon the Proportion which the Heighth F G bears to the Depth of the Body I E, where it appears probable that the Heighth F G does never ſurpaſs half the Depth I E; and with reſpect to the Caſe where the Center of Gravity G falls under the Surface of the Water, this Depth will be always leſs than $\frac{1}{3}$ I E : Thus, by ſuppoſing the Center of Gravity G to be ſituated between thoſe Limits, we ſhall have the following Hypotheſis for both Caſes.

For the Caſe where $\alpha = 0, 8$ and $\beta = 0, 8$.

1ſt. If $FG = 0, 5.\ IE$, we ſhall have
$\overline{CD}^2 > 11, 34.\ \overline{IE}^2$, and therefore
$CD > 3, 37.\ IE$.

2d. If $FG = 0, 4.\ IE$ we ſhall have
$\overline{CD}^2 > 10, 14.\ \overline{IE}^2$, and therefore
$CD > 3, 19.\ IE$.

3d. If $FG = 0, 3.\ IE$, we ſhall have
$\overline{CD}^2 > 8, 94.\ \overline{IE}^2$, and therefore
$CD > 2, 99,\ IE$.

4th. If $FG = 0, 2.\ IE$, we ſhall have
$\overline{CD}^2 > 7, 74.\ \overline{IE}^2$, and therefore
$CD > 2, 79.\ IE$.

5th. If $FG = 0, 1.\ IE$, we ſhall have
$\overline{CD}^2 > 6, 54.\ \overline{IE}^2$, and therefore
$CD > 2, 56.\ IE$.

6th. If $FG = 0, 0.\ IE$, we ſhall have
$\overline{CD}^2 > 5, 34.\ \overline{IE}^2$, and therefore
$CD > 2, 32.\ IE$.

7th. If $FG = -0,1.IE$, we ſhall have
$\overline{CD}^2 > 4,14.\ \overline{IE}^2$, and therefore
$CD > 2,04.\ IE$.

8th. If $FG = -0,2.IE$, we ſhall have
$\overline{CD}^2 > 2,94.\ \overline{IE}^2$, and therefore
$CD > 1,72.\ IE$.

9th. If $FG = -0,3.IE$, we ſhall have
$\overline{CD}^2 > 1,74.\ \overline{IE}^2$, and therefore
$CD > 1,32.\ IE$.

For the Caſe where $\alpha = 0,7$. and $\beta = 0,7$.

1ſt. If $FG = 0,5.IE$, we ſhall have
$\overline{CD}^2 > 10,94.\ \overline{IE}^2$ and therefore
$CD > 3,31.\ IE$.

2d. If $FG = 0,4.IE$, we ſhall have
$\overline{CD}^2 > 9,74.\ \overline{IE}^2$, and therefore
$CD > 3,13.\ IE$.

3d. If $FG = 0,3.IE$, we ſhall have
$\overline{CD}^2 > 8,54,\ \overline{IE}^2$, and therefore
$CD > 2,93.\ IE$.

4th. If $FG = 0,2.IE$, we ſhall have
$\overline{CD}^2 > 7,34.\ \overline{IE}^2$, and therefore
$CD > 2,71.\ IE$.

5th. If

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5th. If $FG = 0, 1. IE$, we ſhall have
$\overline{CD}^2 > 6, 14. \overline{IE}^2$, and therefore
$CD > 2, 48. IE$.

6th. If $FG > 0, 0. IE$, we ſhall have
$\overline{CD}^2 > 4, 94. \overline{IE}^2$, and therefore
$CD > 2, 23. IE$.

7th. If $FG = -0, 1. IE$, we ſhall have
$\overline{CD}^2 > 3, 74. \overline{IE}^2$, and therefore
$CD > 1, 94. IE$.

8th. If $FG = -0, 2. IE$, we ſhall have
$\overline{CD}^2 > 2, 54. \overline{IE}^2$, and therefore
$CD > 1, 60. IE$.

9th. If $FG = -0, 3. IE$, we ſhall have
$\overline{CD}^2 > 1, 34. \overline{IE}^2$, and therefore
$CD > 1, 16. IE$.

§ 71.

This Conſideration will therefore furniſh us with one of the moſt important Rules in the Conſtruction of Veſſels, which is to give a proper Proportion between the Breadth of the Body and its Depth, when the Heighth of the Center of Gravity G is known; and we ſee that whilſt the Center of Gravity G is ſituated above the Water, the Breadth of the Veſſel CD muſt always ſurpaſs the double of the Depth IE; and that in proportion as the Center of Gravity is more elevated.

vated. But as we have here only aſſigned the Limit which the Breadth C D muſt neceſſarily exceed, we ought to enquire how much it muſt exceed this Limit. As that depends on the Violence of the Efforts which a Veſſel muſt ſuſtain, it is abſolutely neceſſary to have recourſe to Experience: Let us ſuppoſe, for example, that a Veſſel may ſail in great ſecurity, the Breadth of whoſe Body C D is to its Depth I E as 5 to 2, or that $CD = 2,5. IE$, and that its Center of Gravity is ſituated preciſely upon the Surface of the Water or $FG = 0$: This being granted, ſince our firſt Caſe gives $CD > 2,32. IE$, which differs from that of Experience by 0, 18, or nearly the thirteenth Part of our Limit, therefore, by augmenting each of our Limits one thirteenth Part, we ſhall find the true Value of the Breadth C D: But the Limits for the other Caſe will require to be increaſed one eighth Part.

§ 72.

With reſpect to the Depth I E of the Body, it is here proper to remark, that Builders commonly give to Veſſels a little more Depth towards the Stern than towards the Head; in this Caſe it is evident that our Depth I E muſt have a certain Mean. The common reaſon given for ſuch a Conſtruction is, that Veſſels are then managed more eaſily; but the true reaſon is without doubt, that when the Veſſel is under Way puſhed by the Wind, its Keel then becomes horizontal; ſince the Force of the Wind commonly gives to the Veſſel an Inclination towards the Head. And with regard to all that we have yet ſaid we obſerve, that beſides increaſing the Section of the Water, the moſt effectual means to augment the Stability, is to carry the Center of Gravity G as low as it is poſſible, or that Circumſtances will admit.

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Upon the Rolling and Pitching of Veſſels.

§ 73.

WHENEVER a Veſſel has received an Inclination from its State of Equilibrium, whatever may be the Cauſe which produced it, it will be urged by its Stability to return to an Equilibrium, and that by an accelerated Motion; from whence it neceſſarily happens that it will paſs beyond it to an oppoſite Inclination, until this Motion ſhall be deſtroyed, by which means the Veſſel will again return to its State of Equilibrium, and will vibrate in the ſame manner as a Pendulum makes its Oſcillations. This Motion will alſo be equally regular, unleſs it be diſturbed by the Reſiſtance of the Water, of which we ſhall not here make any Account. Therefore, ſince theſe Vibrations of a Veſſel round any Axe whatever to which the Inclination has been made, are perfectly like the Oſcillations of a Pendulum, the Nature of them cannot be better known than by determining the Length of a ſimple Pendulum, which would make its Oſcillations in equal Times. Such a Pendulum is named Iſochronal to the Vibrations of a Veſſel.

§ 74.

But having found the Length of ſuch a ſimple Pendulum, which we ſhall put $= l$, the Principles of Mechanics have furniſhed the following Rule for determining the Time of one Oſcillation. At firſt it is neceſſary to know the diſtance which a Body deſcends freely in one Second of Time. This is found by Experiment to be nearly 16 Feet Engliſh Meaſure, which we ſhall here denote by the Letter g, then ſuppoſing the Letter π to denote the Circumference of a Circle whoſe Diameter is Unity, the Time of ſuch an Oſcillation expreſſed in Seconds will be always $= \pi\sqrt{\frac{l}{2g}}$, or we muſt divide the Length of the Pendulum l by double the Heighth g, and multiply the Square Root of this Quotient by

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by π, or by $\frac{22}{7}$, according to the Rule of Archimedes, then we ſhall have the Time of one Oſcillation expreſſed in Seconds.

§ 75.

It is therefore neceſſary to find for each propoſed Caſe this Length l of an Iſochronal Pendulum; which, without doubt, is a Queſtion that requires the moſt profound Reſearches: But for our purpoſe it will be ſufficient to relate here, ſimply the reſults which are derived from the Principles of Mechanics. As we ſhall here conſider the Queſtion in general, when a Veſſel is inclined round any Horizontal Axe whatever paſſing through its Center of Gravity; it is above all neceſſary to know the Stability of the Veſſel with reſpect to this Axe, which is always a Product of the Weight M of the Veſſel by a certain Line, which we ſuppoſe $= s$, by this means the Stability will be $= Ms$; beſides it is alſo neceſſary to know what in Mechanicks is called the Momentum of Inertie of the Veſſel with reſpect to the ſame Axe, which is found by multiplying every particular Part of the Maſs of the Veſſel, each by the Square of its Diſtance from this Axe, and by collecting all theſe Products into one Sum, which we ſhall expreſs by $M.rr$, ſeeing that it will be always a Product of the entire Weight M of the Veſſel, multiplied by the Square of a certain Line, which we here put $= r$. But theſe two Elements being known, the Length of the Iſochronal Pendulum will always be found, by dividing the Momentum of Inertie M. rr by the Stability M S, therefore we ſhall have $l = \frac{rr}{s}$.

§ 76.

After theſe general Determinations we ſhall conſider the Caſe where a Veſſel makes its Vibrations round the great Axe directed from Head to Stern; this Motion is named Rolling, and by which the Veſſel is alternately inclined from one Side to the other: It is at firſt neceſſary to remark, that this Motion may be preſerved a long Time in a calm Sea, ſince the Body of the Veſſel is in general ſo much curved round this Axe, its Motion can meet with little Reſiſtance in the Water, whoſe Efforts being directed very nearly towards the Axe itſelf, will produce but little Momentum capable of diſturbing this Motion. It will therefore be eaſy to obſerve the Time of theſe Vibrations, and by this means we ſhall find by one Experiment only the Length of the Iſochronal Pendulum $l = \frac{rr}{s}$, ſo that from thence we might aſſign the Value of one of theſe two

Quantities r or s, the other being already known. Beſides we alſo perceive, that this Motion of Rolling will be ſo much the more ſlow and gentle in proportion as the Iſochronal Pendulum becomes longer; from whence it is evident, that, ſince it is not proper to diminiſh the Denominator s, it will be neceſſary to uſe the means of increaſing the Numerator $r\,r$ as much as circumſtances will permit. We ſhall obtain therefore this augmentation of the Length l by placing all the Weight of the Cargo as far diſtant as poſſible from the great Horizontal Axe, which paſſes through the Center of Gravity G according to the Length of the Veſſel.

§ 77.

The Vibrations made round the Tranſverſe Axe of the Veſſel are very near the ſame; theſe we call Pitching; by which the Veſſel is alternately inclined towards the Head and Stern. But then in the expreſſion $l = \frac{r\,r}{s}$, the Denominator s is much greater than in the preceding Caſe, ſince the Stability with reſpect to this Axe is many times greater than that which correſponds to the other, from whence the Value of l would become much leſs, and therefore the Motion of the Pitching more rapid. But we muſt likewiſe conſider, that the

Value of the Letter *r* is in this Caſe much greater than in the preceding one, ſeeing that all the Weights contained near the Head and Stern are very diſtant from the Tranſverſe Axe; and this circumſtance might occaſion as great a Value for the Letter *l* as in the preceding Caſe: But this Motion of Pitching could not continue ſo long as that of Rolling, becauſe the Head and Stern, on account of their Obliquity, would ſuffer a very conſiderable Reſiſtance by Diping and Riſing alternately, therefore this Motion will in general be very ſoon deſtroyed, the Water being ſuppoſed perfectly calm.

§ 78.

But when the Sea is much agitated, we eaſily conceive that both the Rolling and Pitching muſt ſuffer very conſiderable alterations from it, ſeeing that the alternate riſing and falling of the Waves themſelves are capable of producing a Vibration in the Veſſel, altho' it had not received an Inclination from any other Force. But in order to determine the Motions with which the Veſſel would then be impreſſed, Theory leaves us entirely in the dark, ſeeing that on one ſide we are ſtill abſolutely ignorant of the Laws by which Water agitated acts upon Bodies which float therein, and on the other hand the Expreſſion before found, for the Stability could no longer have place for the ſame reaſon; and therefore alſo the Length of the Iſochronal Pendulum, which we have juſt aſſigned, becomes entirely falſe. Alſo experience has ſhewn us that the Forces which a troubled Sea exerts by its Waves upon a Veſſel are entirely different from thoſe which we obſerve in a Calm. We even remark, that when a Veſſel is lifted up by the Waves, it riſes by an accelerated Motion, and falls by a retarded one; which appears directly oppoſite to the Principles that we commonly form upon the action of Water.

§ 79.

However, altho' we are very far from being able to determine any thing with certainty upon this matter, it will not be improper to remark, that the Waves commonly ſucceed each other in a manner ſufficiently regular and by equal intervals of Time, ſo that when a Veſſel has received the firſt Stroke, it will receive at equal intervals of Time the ſecond, third, &c. Shocks. Therefore if the Veſſel itſelf was of ſuch a nature as to make its Vibrations in the ſame intervals of Time, the following Stroke of the Waves would always meet the Veſſel in the ſame ſituation as the preceding one, and therefore its Force would be re-united for to augment the Motion of the Veſſel, by which means it might

Plate I. Book 1.

Fig: 1.

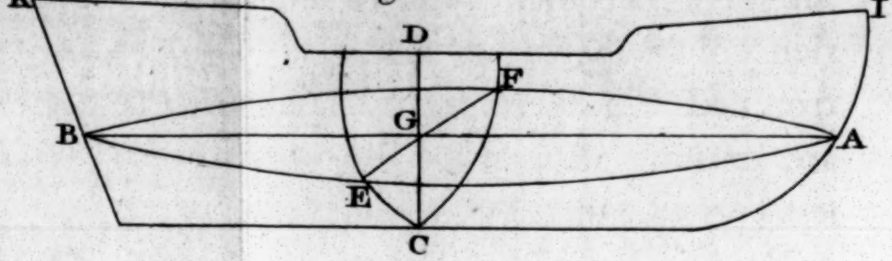


Fig: 2.

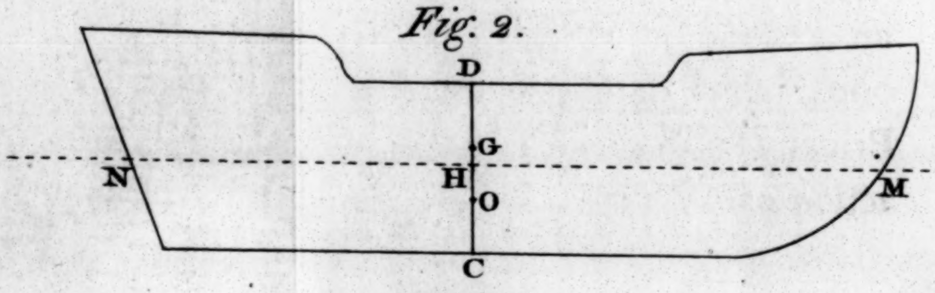
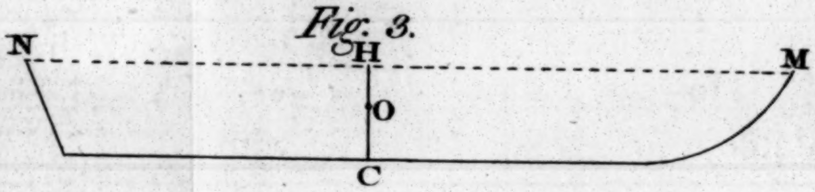


Fig: 3.



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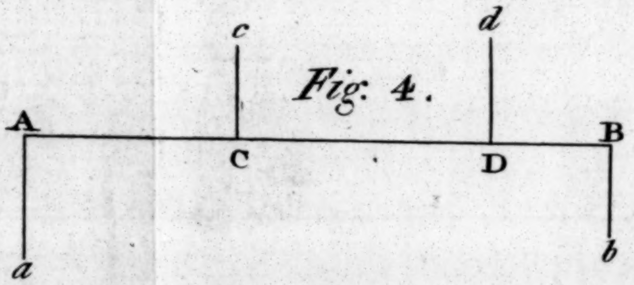


Fig. 5.

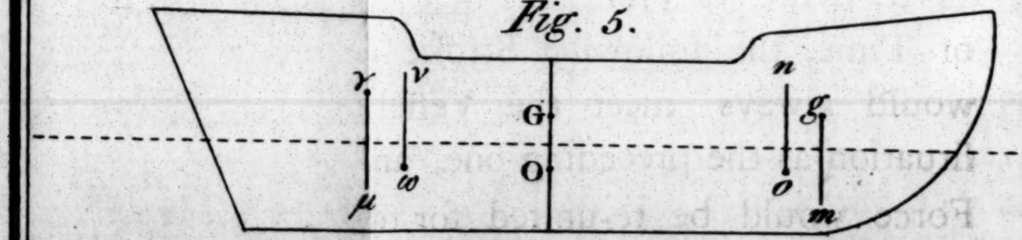


Plate II. Book 1.

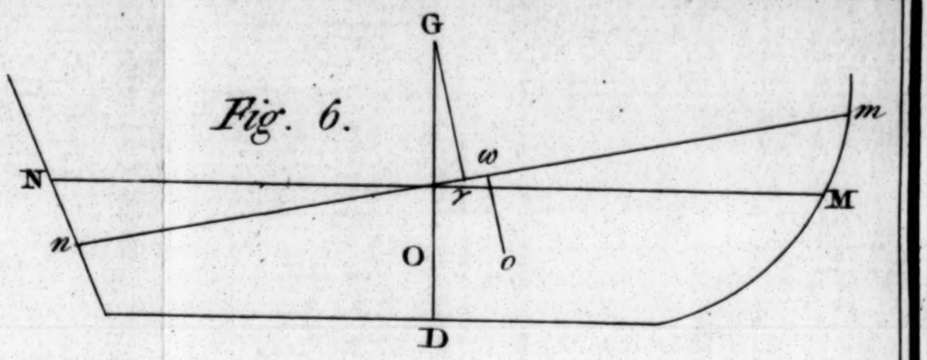


Fig. 6.

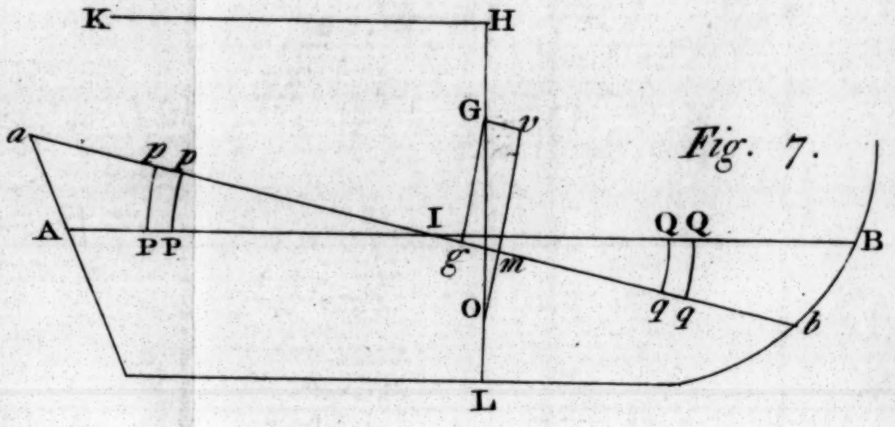


Fig. 7.

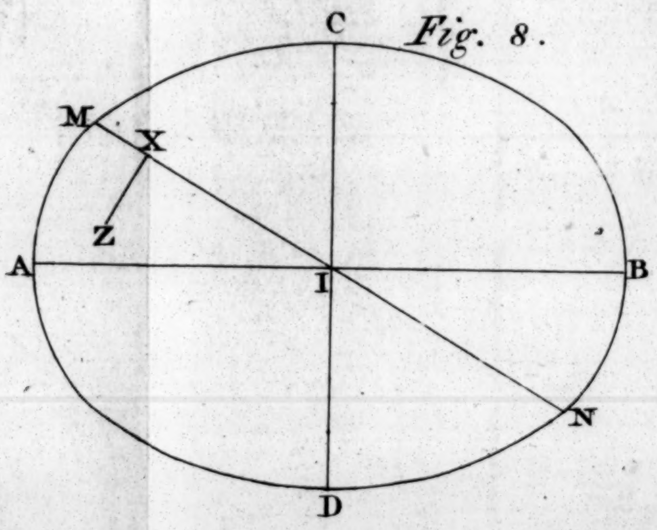


Fig. 8.

Plate III. Book 1.

Fig: 9.

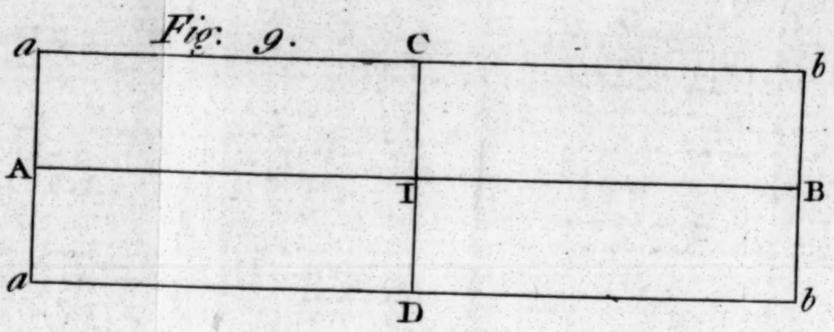


Fig. 10.

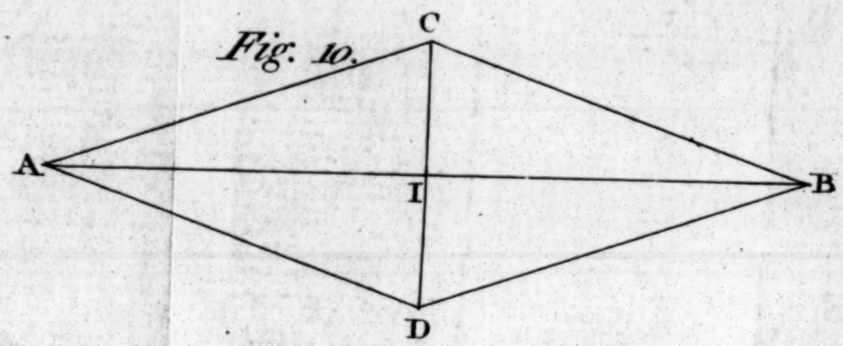
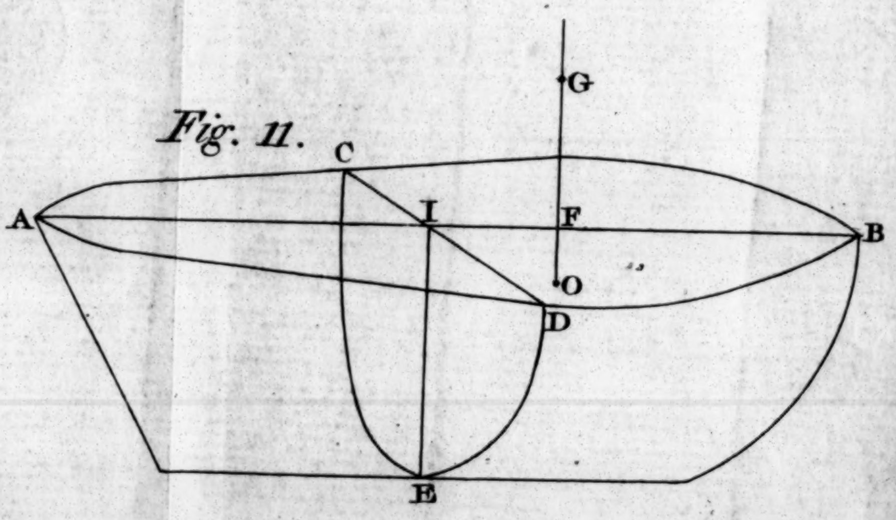


Fig. 11.



CHAP. X.

at laſt become dangerous. But if the ſaid intervals of Time between the ſucceſſion of the Waves, and the Vibrations of the Veſſel, were ſo proportioned among themſelves, that the following Stroke ſhould deſtroy the Effect of the preceding one, the Veſſel might then ſuffer very rude Shocks; more eſpecially in Pitching, when the Head and Stern had received very quick Motions, and that new Efforts of the Water were ſuddenly oppoſed to them, as there might then well reſult ſuch a tremulous Motion in all Parts of the Veſſel as would endanger a loſs of the Maſts.

BOOK II.

Of the Reſiſtance which Veſſels experience in their Courſes, and upon the Action of the Rudder.

CHAP. I.

Upon the Reſiſtance of a Plane Surface moving in Water.

§ 1.

As long as a Body immerſed in Water continues at reſt, every Part of its Surface ſuſtains equal Preſſures; which being reſolved into Horizontal and Vertical ones, the former mutually deſtroy each other, and the Vertical ones are reduced to a Force equal to the Weight of a Volume of Water, which would occupy the ſame Space as the Part immerſed, and by which it is puſhed upwards in a vertical Manner, as we have already demonſtrated. But as ſoon as the Body is put in Motion, it will then experience another Force beſides theſe Preſſures, and which is oppoſed to its Motion; this we name the Reſiſtance of the Water, and which we propoſe here to determine: At firſt we muſt obſerve, that this Force of Reſiſtance depends principally on the Figure of the Body, whilſt the Preſſures before-mentioned are abſolutely independant: for which reaſon it is proper to begin our Reſearches by the conſideration of Plane Surfaces, which are moved in Water with a certain Velocity, both in a direct and oblique Manner. A Surface is moved in a direct Manner in Water when the Direction of its Motion is perpendicular to the ſame Surface; but obliquely, if this Direction is oblique to it.

§ 2.

Plate I. Fig. 1.

Let us therefore conſider any Plane Surface whatever moving in Water, according to the Direction E F perpendicular to the ſame Plane, with a Velocity which we ſuppoſe $= c$, denoting the Space gone over in one Second of Time, as this manner of repreſenting Velocities will give us the moſt clear idea of them: then, ſince that Surface could not continue its Motion without puſhing the Water before it, there will happen a Colliſion; from whence muſt neceſſarily reſult a certain Force by which the Surface will be impeded or urged backwards; and we ſee evidently that this Force will be perpendicular to the Surface, and therefore directly contrary to its Motion; or rather, this

Surface will be in the ſame Situation, as if being puſhed Horizontally it had to ſuſtain a certain Column of Water; ſo that if we knew the Heighth of this Column we ſhould have an exact knowledge of the Reſiſtance which this Surface meets with in the Water. It is required therefore to find this Column, whoſe Heighth will furniſh us with the juſt Meaſure of the Reſiſtance which we ſeek; for we have only to multiply this Heighth by the Area of the Surface itſelf, in order to find the Solidity of a Maſs of Water whoſe Weight would be preciſely equal to the Force of the Reſiſtance.

§ 3.

But the following Reaſoning will lead us to the knowledge of this Heighth: It is at firſt evident, that the Surface A B C D, by moving in Water with a Velocity = *c*, will ſuſtain the ſame Effort as if the Surface was at reſt, and that the Water ſtruck it in a perpendicular manner with the ſame Velocity: but if in this Situation ſome Part of the Surface was pierced by a ſmall Hole, the Water would eſcape there preciſely with a Velocity = *c*. Let us at preſent give to the ſaid Column of Water ſuch a Heighth, ſo that the Water might run out with the ſame Velocity at a Hole made in the Bottom; there cannot then be any doubt but the Surface will in both Caſes, be equally preſſed or acted upon; therefore putting this Heighth $= h$, and the Area of the Baſe, or the Surface $= f^2$, the Solidity of the Column will be $= f^2 h$, and conſequently the Weight of an equal Volume of Water will give us the true force of the Reſiſtance, by which the Surface will be preſſed or acted upon in a perpendicular Manner, or in a Direction contrary to that of its Motion.

§ 4.

But we know, both from Theory and Experience, that the Water contained in a Veſſel whoſe Heighth is $= h$, will run out through a Hole in the Bottom with the ſame Velocity that a Body falling from the ſame Heighth h would acquire: And if the Letter g denotes the Heighth through which a Body falls in one Second, we alſo know the Velocity will be ſuch, that it would run through a diſtance $= 2\sqrt{gh}$ in the ſame Time. Since therefore this Velocity is ſuppoſed $= c$, or $2\sqrt{gh} = c$, by taking the Square $4gh = c^2$, from whence we have the Heighth ſought $h = \frac{c^2}{4g}$: conſequently the Force of the Reſiſtance which the ſuppoſed Plane Surface A B C D $= f^2$ will experience by moving in Water with the Velocity c, will be $= \frac{c^2 f^2}{4g}$, and by this Force the Surface will be acted upon in a Direction contrary to its Motion: Hence we ſee that this Reſiſtance is always proportional to the Square of the Velocity, and alſo proportional to the Area of the Surface itſelf, ſo that by this means the Reſiſtance is perfectly determined.

§ 5.

Plate I. Fig. 2. Let us now, in the ſame manner conſider the Caſe where the Surface A B C D is moved in Water with the ſame Velocity $= c$, but in an oblique Direction to its Plane, and ſuppoſe this Inclination to be any Angle whatever $= \varphi$. Let A B repreſent the propoſed Plane Surface, E F the Direction of its Motion with a Velocity $= c$, and the Angle A E F $= \varphi$; it is then evident that this Plane will ſuſtain the ſame Force as if it was at reſt, and the Water ſtruck it in the Direction F E, with the ſame Velocity: this Force might therefore ſtill be compared with the Weight of a certain Column of Water ſuſtained by the ſame Baſe; and as this Force is properly but a mere Preſſure, it follows, that it acts perpendicularly upon the Surface A B in the Direction E G, which will not therefore be directly contrary to the Direction of the Motion E F; but having found this Preſſure E G, which ſuppoſe $= P$, we have only to reſolve it into two others;

one in the Direction E H, the ſame as E F, the other in that of G H, which is perpendicular thereto, and conſequently does not oppoſe itſelf to the Motion. From whence the Reſiſtance, directly contrary to the Motion, will be = P. Sin. ϕ; ſince the Angle E G H is evidently equal to the Angle A E F = ϕ, and becauſe E H repreſents the Sine of that Angle, Radius being = E G. But in order to find this Preſſure, let us apply the ſame Principle which we have juſt made uſe of in the preceding Caſe, and conſidering a Current of Water to ſtrike upon the Surface A B at reſt in the Direction F E, with a Velocity = c, it is plain, that if the Surface was pierced with a Hole at E, the Water would freely paſs there in the Direction E H, without changing either its Direction or Velocity; let us now reſolve this Velocity into two others, one in the Direction E I, perpendicular to the Surface, and the other into that of I H parallel thereto; then, ſince this laſt has no influence upon the Motion, the firſt only in the Direction E I muſt be conſidered as the effect of the Shock which the Surface experiences. But becauſe the Angle E H I = A E F = ϕ, this Velocity in the Direction E I will be = c. Sin. ϕ: it is required therefore to find the Heighth of a Column of Water, which being ſuſtained by the ſame Baſe, the Water would fly out with a Velocity = c. Sin. ϕ: But

Plate I. Fig. 3.

CHAP. I.

But we have juſt ſhewn that, by calling this Heighth b, we might put $2\sqrt{bg} = c.\,\text{Sin.}\,\varphi$; from whence we have $b = \frac{c^2.\,\text{Sin.}\,\varphi^2}{4g}$, which being multiplied by the Area of the Surface f^2, will give the force of the Reſiſtance in the Direction E G perpendicular to the Surface, and is the total Reſiſtance that we have called P; therefore the Reſiſtance in a Direction contrary to the Motion will be $\text{P. Sin.}\,\varphi = \frac{c^2 f^2.\,\text{Sin.}\,\varphi^3}{4g}$.

Fig. 2.

§ 6.

Thus, by Reaſonings both ſimple and concluſive, we have obtained Expreſſions which ſhow us in all Caſes where a Plane Surface $\text{A B} = f^2$ is moved in Water with a Velocity $= c$, and with an Obliquity $\text{AEF} = \varphi$, both the total Reſiſtance, $\frac{c^2 f^2.\,\text{Sin.}\,\varphi^2}{4g}$, as well as that in a Direction contrary to the Motion $= \frac{c^2 f^2.\,\text{Sin.}\,\varphi^3}{4g}$. From the firſt we ſee that the total Reſiſtance is in a Ratio compoſed, 1ſt, of the Area f^2 of the Surface itſelf; 2dly, of the Square of the Velocity c^2; and 3dly, of the Square of the Sine of the Obliquity, or of the Angle $\text{AEF} = \varphi$. But the Reſiſtance in a Direction contrary to the Motion is in a Ratio compoſed, 1ſt, alſo of the Area f^2, and, 2dly, of the Square of the Velocity c^2;

but 3dly, of the Cube of the Sine of the Obliquity. It is upon theſe two Principles, known long ago to Mathematicians, that we have founded all the Theory of Reſiſtance which ſolid Bodies moving in any Fluid whatever have to overcome. We have alſo heretofore given different Demonſtrations of theſe Principles; but that which has been juſt delivered appears to be the moſt evident and concluſive.

§ 7.

Plate I. Fig. 4.

In order that this Matter may be the better explained, let us compare the two following Caſes together. 1ſt, Let A B be a Plane Surface $= f^2$, moved through the Water in a direct manner A a with a Velocity $= c$; and 2dly, that another Surface A C is moved in the ſame Direction with the ſame Velocity $= c$, but obliquely, the Angle of Obliquity being a A C $= \varphi$. Let the Area of the Surface A C be to the Area of the firſt A B as the Hypotheneuſe A C is to the Baſe A B, then having drawn the Line B b parallel to A a, in order that theſe two Surfaces may oppoſe the ſame Column of Water; then the Area of this laſt Surface A C will be $= \frac{f^2}{\text{Sin.}\ \varphi}$. This being done, ſince the Reſiſtance of the Surface A B has been found $= \frac{c^2 f^2}{4 g}$ and as the Reſiſtance of the Surface A C, is contrary to the Motion, by ſubſtituting $\frac{f^2}{\text{Sin.}\,\varphi}$ inſtead of f^2 in our laſt Expreſſion, it will be $= \frac{c^2 f^2 .\, \text{Sin.}\,\varphi^2}{4g}$; therefore theſe two Reſiſtances are to each other as Unity to the Square of the Sine of the Obliquity. Hence, if the Angle B A C was $= 45°$, becauſe $\varphi = 45°$ and Sin. $\varphi^2 = \frac{1}{2}$, the Reſiſtance of the Hypotheneuſe A C would be preciſely one half of that of the Surface A B, and if we ſhould make the Angle B A C $= 60°$, becauſe that of $\varphi = 30°$, and the Sin. $\varphi = \frac{1}{2}$, the Reſiſtance of A C would become four times greater than that of A B. And in general, the more we increaſe the Angle B A C the leſs will be the Reſiſtance of the Surface A C, and will at laſt almoſt entirely vaniſh. For taking the Angle B A C $= 80°$, ſo that the Angle $\varphi = 10°$, the Reſiſtance of A C will be only $\frac{1}{33}$ part of that of A B; and if this Angle B A C be taken $= 85°$, or $\varphi = 5°$, the Reſiſtance will be reduced to the $\frac{1}{131}$ part of it. From hence we perceive already how the Reſiſtance of Veſſels may be conſiderably diminiſhed by lengthening and contracting the Fore Part.

§ 8.

In this Compariſon we have only conſidered the Reſiſtance which is directly contrary to the Motion: This, without doubt, is ſufficient, when it is required to determine the Force of the Reſiſtance that Veſſels experience in their Courſes; and we know already what method we muſt purſue in order to find this Reſiſtance, whatever may be the Figure of the Veſſel. For we have only to divide the Surface of the Body into ſeveral very ſmall Parts, which we may conſider as Plane Surfaces, and to ſearch the Obliquity of each with reſpect to the Direction of the Motion. But it is likewiſe equally neceſſary to determine in all Caſes what Force the Reſiſtance exerts for to incline the Veſſel; and then it is neceſſary to make uſe of our principal Expreſſion, which denotes the total Reſiſtance, which was $\frac{c^2 f^2 . \text{Sin.}\,\varphi^2}{4g}$, in order that we may by the reſolution of Forces deduce thoſe which are able to produce any Inclination whatever. Finally it is here neceſſary to remember, that the Letter g expreſſes a Length of 16 Engliſh Feet, or very near, which is ſufficiently exact for theſe kind of Reſearches.

CHAP. II.

Upon the Reſiſtance of Veſſels in their Direct Courſes.

§ 9.

WHEN the Motion of a Veſſel through the Water is parallel to its Keel, or rather to the great Axe of its Body, the Courſe is called Direct, in order to diſtinguiſh it from thoſe whoſe Directions are inclined to that of the Keel. It is therefore neceſſary that the Direct Courſe ſhould be firſt conſidered; and in order to determine the Reſiſtance which the Veſſel will then experience, we have only to conſider the immerſed Part or the Body of the Veſſel, whoſe diametrical Section let be repreſented by A B C D, its great Axe A B will at the ſame time expreſs the Direction of Motion; the Velocity of the Veſſel we ſhall always ſuppoſe $= c$, or the Space run through with that Velocity in one Second of Time. Let A be the Head, B the Stern, and C D the Keel; alſo ſuppoſe G to be the Center of Gravity of the whole Veſſel, from whence let the vertical Line G E be drawn, cutting the Water-Section in F, and the Keel at E.

Plate I. Fig. 5.

§ 10. Let

§ 10.

Let us at firſt conceive, that the greateſt tranſverſe Section of the Veſſel ſtrikes the Water with a Velocity $= c$, and that the Direction of its Motion is perpendicular to this Section; this would be the caſe if the Veſſel was of a priſmatic Form, and all the tranſverſe Sections equal to each other; the Head would then be terminated at A by a vertical Plane A *a*, perpendicular to the Axe A B, and equal to the tranſverſe Section F E, which we ſhall always conſider as the greateſt. Let the Area of this Section A *a* be $= f^2$, then ſince it ſtrikes againſt the Water in a direct manner, the Reſiſtance oppoſite to the Direction of Motion will be $= \frac{c^2 f^2}{4g}$.

§ 11.

Next it is evident that the mean Direction of this Reſiſtance will paſs through the Center of Gravity of the Plane Surface A *a*, which being ſuppoſed at *c*, the horizontal Line *c d* parallel to A B will ſhew the Direction of the Force equivalent to the whole Reſiſtance that the Veſſel meets with in its direct Courſe; from whence we conclude that the Veſſel will, in the firſt place, be retarded by the Force $\frac{c^2 f^2}{4g}$, and ſecondly,

CHAP. II.

ſince the Direction $c\,d$ does not paſs through the Direction of the Center of Gravity G, this ſame Force will alſo produce a Momentum tending to incline the Veſſel round its tranſverſe Axe paſſing through the Point G, and perpendicular to the diametrical Plane repreſented in the Figure: this Momentum will therefore be $= \frac{c^2 f^2}{4g} \times G\,d$, by which the Veſſel will be inclined towards the Head A, and will in conſequence of it be more immerſed in the Water. Now, as we know the Stability of the Veſſel with reſpect to the ſame Axis which is $= M\,s$, we may even aſſign the Angle i, which the Veſſel will make by its Inclination. For this purpoſe we muſt conſider that ſince M denotes the Weight of the whole Veſſel, and as our Momentum of Force is expreſſed by a Volume of Water, this Force muſt in the firſt place be reduced to an abſolute Weight; wherefore, if we ſuppoſe the whole Volume of the immerſed Part, as above $= V$, the Momentum of Force will be $= \frac{M}{V} . \frac{c^2 f^2}{4g} . G\,d$. which being divided by the Stability Ms, will give the Sine of the Inclination or

$$\text{Sin.}\, i = \frac{c^2 f^2}{4gV} . \frac{G\,d}{s}.$$

§ 12.

This Reſiſtance which the greateſt tranſverſe

Section experiences from the Water when moved with an uniform Velocity, may be conſidered as a Standard whereby the Reſiſtance may alſo be eſtimated when the Fore Part of the Veſſel is of any particular Form, by ſhewing how much the actual Reſiſtance is leſs than that of the tranſverſe Section; and from what has been ſaid before, we readily conclude that the actual Reſiſtance may become much leſs than that which the tranſverſe Section would experience, ſince we have demonſtrated that the Reſiſtance decreaſes as the Action of the Water upon the Head becomes more oblique; from whence it follows, that the more the Head of a Veſſel is lengthened and continually narrowed before, the more alſo will its Reſiſtance be diminiſhed. But as the Contraction uſually takes place not only on the Sides towards the Middle, but likewiſe from the lower to the upper Part, we eaſily comprehend a Force will reſult therefrom, by which the Veſſel will be elevated, and its Motion retarded; it is therefore neceſſary both theſe Forces ſhould be conſidered in order to obtain a clear and juſt idea of the whole Effect which the Reſiſtance is capable of producing.

§ 13.

But as an Inveſtigation of theſe Forces requires Calculations that are extremely difficult even for Veſſels of the moſt ſimple Figures; and as we are obliged to content ourſelves with

CHAP. II.

mere Approximations, before we enter into a more particular Detail, it will be proper to take a general View of the Subject. Let therefore A B C D be the Diametrical Section of the Veſſel, or rather of the Part immerſed, as before, and let the Line A C repreſent the riſing of the Stem from the Keel C to the extremity A at the Surface of the Water; or rather let the ſame Line A C repreſent the Rake of the Veſſel, and ſuppoſe the Velocity with which the Veſſel runs in the direction B A to be $= c$; it appears evident upon the firſt View that all the Efforts of the Reſiſtance may be reduced, 1ſt, to a horizontal Force in the Direction c P, and therefore directly contrary to the Motion of the Veſſel; this we ſhall expreſs by the letter P. 2dly, to a vertical Force whoſe Direction let be ſuppoſed that of d Q and which we ſhall repreſent by the letter Q; it is alſo proper to remark that the Interſection R, which may be conſidered as the Center of the Reſiſtance through which the Direction of the Force R S, equivalent to the two preceding Forces, will paſs; ſo that all the Efforts of the Reſiſtance are reducible to the ſingle Force R S, whoſe Quantity is known to be $= \sqrt{P^2 + Q^2}$; and the Tangent of the Angle P R S or its Inclination to the Horizon will be $= \frac{Q}{P}$; by this means it will

Plate I. Fig. 6.

be ſufficient to conſider the two Forces P and Q, each of which is always proportional to the Square of the Velocity of the Veſſel.

§ 14.

At preſent we ſhall conſider the Effect which each of theſe Forces will produce on the Veſſel: Now the firſt or horizontal Force P, will (as in the preceding caſe) produce a double Effect, one which is directly oppoſite, and tending to retard the Motion of the Veſſel in the ſame manner as if it was applied to the Center of Gravity G; the other Effect ariſes from the Momentum of this Force with reſpect to the tranſverſe Axe, which being = P . G P will tend to depreſs the Head of the Veſſel. In the ſame manner the other Force Q which is Vertical, will produce a double Effect, one urging the Veſſel directly upward in the ſame manner as if it was applied to the Center of Gravity G, and by this means the whole Weight of the Veſſel will be diminiſhed by the Quantity or Weight Q; and the other Effect of this Force will produce a Momentum with reſpect to the ſame Tranſverſe Axe = Q . F Q, whoſe Direction being contrary to that of the firſt Momentum P . G P will tend to elevate the Head: therefore if this Force be greater than the firſt, the Stern of the Veſſel will be depreſſed by a

Momentum of Force $= Q . FQ - P . GP$, which being divided by the Stability of the Veſſel with reſpect to the ſame tranſverſe Axe will give the Sine of the Inclination ariſing from the Action of theſe two Forces.

§ 15.

From what has been ſaid it is evident that, in order to preſerve the Veſſel in the Motion which we have ſuppoſed with a Velocity $= c$, it muſt be urged directly forward by a Force P, equivalent to that which it experienced from the Reſiſtance; and ſince the Weight of the Veſſel M is diminiſhed on account of the Reſiſtance by a Weight $= Q$, we have only to add a new Weight Q placed at the Center of Gravity G itſelf in order that the ſaid Center may not be changed. Laſtly, in order that the Veſſel may ſuffer no Inclination we muſt apply the firſt Force P above the Center of Gravity G, as at H, ſo that its Momentum $P . GH$ may become preciſely equal to that by which the Veſſel is inclined towards the Stern, which was $Q. FQ - P. GP$, therefore

$$P. GH = Q. FQ - P. GP; \text{ conſequently}$$

$$GH = \frac{Q}{P} . FQ - GP, \text{ and}$$

$$PH = \frac{Q}{P} . FQ.$$

Now let the Force $HK = P$ be applied to the Veſſel, and it is evident that the Point H will be preciſely in the Interſection of the Vertical Axe E G with the true Direction of the Reſiſtance R S.

§ 16.

But in practice it is not neceſſary to load the Veſſel with a new Weight Q, ſince it is very advantageous for the Reſiſtance to diminiſh the Weight of the Veſſel, and it would therefore be highly improper to increaſe it: Laſtly, as the whole Veſſel will in this caſe be ſomething elevated, the Hollow of the Body will become leſs; which is a favourable circumſtance, ſince the Reſiſtance will thereby be ſomewhat diminiſhed, and conſequently a leſs Force will then ſuffice to continue the Veſſel in its Motion.

CHAP. III.

Upon the Method of eſtimating the Reſiſtance of the Fore Part of a Veſſel whoſe Figure is given.

§ 17.

IF the Fore Part be of ſuch a Figure that all the Elements of its Surface are equally inclined to the Direction of Motion, it is eaſy to determine the Reſiſtance which is contrary to the Motion; and with reſpect to the Reſiſtance which a Veſſel experiences in a vertical Direction it may be neglected, as we ſhall ſhew hereafter. Therefore if we put the Area of the greateſt tranſverſe Section of the Body $= f^2$, the Velocity of the Veſſel in the Direction of its principal Axe B A $= c$ and the Angle with which the whole Surface of the Fore Part is inclined to the Direction of Motion $= \varphi$, we have ſeen that the Reſiſtance which is oppoſed to the Motion, is equal to the Weight of a Quantity of Water whoſe Volume would be $= \frac{c^2 f^2 \text{ Sin. } \varphi^2}{4g}$. Now if the Plane formed by the greateſt tranſverſe Section ſhould move directly through the Water with the ſame Velocity, its Reſiſtance would be $= \frac{c^2 f^2}{4g}$; ſo that in the other caſe the Reſiſtance is as much diminiſhed as the Square of the Sine of the Angle φ differs from Unity: and hence it appears that the Reſiſtance might be diminiſhed at pleaſure, if other Circumſtances equally eſſential to the Veſſel did not Limit it.

§ 18.

Now ſuch an Equality of Inclination thro' the whole Surface of the Fore Part of the Veſſel may have place in an infinite number of Caſes, ſome which we ſhall ſhew; as for inſtance, if the greateſt tranſverſe Section be a rectangular Parallelogram, as M N *m n*, and the Head have the Figure of a Wedge terminated by the vertical Line A *a*, ſo that all the horizontal Sections A M N, *a m n* would be Triangles equal among themſelves: then the two Faces A *a* M *m* and A *a* N *n* will ſuſtain all the Efforts of the Water under the ſame Angle F A M = F A N $= \varphi$ whoſe Sine being $= \frac{FM}{AM}$ the Reſiſtance of this Figure will be to that of its Baſe M N *m n* as FM^2 to AM^2; or denoting the Reſiſtance of the Baſe by the letter R, that of the Fore Part will be $= R. \frac{FM^2}{AM^2}$.

Plate I. Fig. 7.

§ 19.

The ſame thing happens if the greateſt tranſverſe Section be a rectangular Parallelogram as before, and the Fore Part E A be an inclined Plane riſing from E to A in ſuch a manner that all the Sections parallel to the diametrical one, become right-angled Triangles equal to A F E: For in this Caſe the Angle of Obliquity would be F A E, and its Sine $= \frac{EF}{AE}$; from whence the Reſiſtance will be $= R. \frac{EF^2}{AE^2}$ where R denotes the Reſiſtance of the principal Section F E. The ſame Reſiſtance would alſo take place if the

Plate I. Fig. 8.

CHAP. III. the greateſt tranſverſe Section was a Semicircle deſcribed with the Radius E F and the Fore Part half a Cone ſituated upon this Baſe, with its Vertex at A. It is evident alſo, that this Reſiſtance is equally applicable to any Pyramid whoſe Baſe is a Polygon circumſcribed about that of the Cone before-mentioned; but as all theſe Figures do not enter into practice, we ſhall ſhew a Method of determining the Reſiſtance when the Fore Part is of any given Figure whatever.

§ 20.

Plate II. Fig. 9. For this purpoſe let C D E repreſent half the greateſt tranſverſe Section, and parallel to it let us conceive ſeveral Sections to be made at given Diſtances from thence to the extremity of the Fore Part of the Veſſel and which we ſuppoſe to be projected upon the Plane of the greateſt tranſverſe Section: of theſe let any one whatever be repreſented by M P Q N, and let $mpqn$ be that of the next adjoining one towards the Head, deſcribed in the ſame manner as Draughtſmen lay down the Lines of a Veſſel: But for our purpoſe it will be ſufficient to conſider theſe Figures, only from the Keel E to the Surface of the Water C D. Let us alſo ſuppoſe ſeveral tranſverſe Lines, as R P p r and S Q q s to be drawn, cutting the former Lines nearly at Right-angles, ſo that by means of

theſe two Orders of Lines the Area of the firſt ſemi Section CDE will be divided into ſeveral ſmall Trapeziums and Triangles nearly Rectangular; as for example, the Trapezium P*p*Q*q* which we muſt take ſo ſmall that the Portion of the Surface correſponding to each may be regarded as a Plane, whoſe Inclination to the Direction of Motion being known, we have only to multiply the Area P*p*Q*q* by the Square of the Sine of this Inclination, and collect into one Sum all theſe Products in order to have the Value of the Expreſſion $f^2 . \text{Sin.} \varphi^2$, the double of which being multiplied by $\frac{c^2}{4g}$ will give the Reſiſtance which the Fore Part of the Veſſel meets with in a Direction contrary to that of its Motion.

§ 21.

In order to accompliſh theſe Operations, let us conſider any one of the aforeſaid Parts PQ*pq* as deſcribed upon the Plane of the Section MN; and ſuppoſing the Interval between this Section and the following one *mn* to be $= k$, let the Perpendiculars $p\pi$ and $q\varrho$ each $= k$ be raiſed upon this Plane, then the Points π and ϱ will be in the Surface of the Fore Part of the Veſſel as well as the Points P and Q: Therefore the Quadrilateral $P\pi\varrho Q$ will repreſent a Portion of the Surface, and will be the ſame which we have

con-

conceived to correſpond to the ſmall Area $P p Q q$ in the preceding Figure. But here the Perpendiculars $p \pi$ and $q \varrho$ being parallel to the great Axe of the Body, and conſequently alſo parallel to the Direction of Motion, we have now only to determine the Angle by which theſe Lines would be inclined to the Surface $P p q Q$. But it is evident that if the Angles $P p q$ and $Q q p$ were right, the Angles $P \pi p$ and $Q \varrho q$ would truly meaſure this Inclination: Therefore ſince we ſuppoſe theſe Angles nearly right, the Deviation will be inſenſible: However ſince it is neceſſary to multiply by the Square of the Sine of the Inclination, we may, when the Angles $P \pi p$ and $Q \varrho q$ are not exactly equal, multiply by the Product of their Sines, which being $\frac{P p}{P \pi}$ and $\frac{Q q}{Q \varrho}$, we ſhall have then for the Part $P Q p q$ this Product $\frac{P Q p q \cdot P p \cdot Q q}{P \pi \cdot Q \varrho}$: and in the ſame manner having found other ſimilar Products, we muſt proceed as directed in the preceding Article.

§ 22.

If a more rigid exactneſs ſhould be required in this Calculation, as it may well happen, that the four Points P, Q, π, ϱ, are not ſituated in the ſame Plane, we might then draw a Diagonal $P \varrho$ or $Q \pi$ in order to have two Triangles, each of which would be a Plane whoſe

Inclinations may be eaſily determined: but ſince it would be almoſt impoſſible to determine in ſuch Caſes to which of the Diagonals $P\varrho$ and $Q\pi$ we ought to give a preference; we cannot therefore flatter ourſelves with any expectation of arriving nearer to the truth than by making uſe of our Rule; beſides, theſe precautions extend only to *Minutiæ*, which in practice can never be of any advantage; and we may always be very well ſatisfied with having found, that the Fore Part of a given Veſſel, for example, meets with a Reſiſtance which is only one tenth part of that which would ariſe from its greateſt tranſverſe Section, although in reality the Proportion ſhould be as 1 to $9\frac{1}{2}$ or as 1 to $10\frac{1}{2}$.

§ 23.

But as the Theory of Reſiſtance which we have hitherto conſidered muſt be allowed to be very defective, and that we cannot entirely depend upon the Concluſions which are drawn therefrom, we may well ſpare ourſelves the trouble of ſuch difficult Reſearches: For although we have already ſuppoſed that the ſimple Preſſures which the Body of a Veſſel ſuſtains when in Motion do mutually deſtroy each other, yet we are certain this can only happen when the Veſſel is at Reſt, ſince the Water behind the Veſſel muſt follow and overtake it before any Preſſure can be exerted, it is therefore evident that the Preſſure

CHAP. III. Preſſure upon the Aft Part cannot be ſo great when the Veſſel is in Motion, as when it is at Reſt, whilſt the Preſſure upon the Fore Part will nearly be the ſame in both caſes: From whence it follows, ſince the Preſſure upon the Fore Part is no longer counterbalanced by that upon the Aft Part, the Effect of this Reſiſtance muſt neceſſarily be increaſed; and this Increaſe will by this means be ſo much the more conſiderable as the Velocity of the Veſſel becomes greater; and however little conſideration we may employ upon this matter, we may eaſily conceive that this Increaſe muſt depend principally upon the Figure of the Aft Part of the Veſſel, which we have hitherto intirely neglected.

§ 24.

On this account it appears very probable that, notwithſtanding all our endeavours to determine the exact Reſiſtance, we may perhaps ſtill vary conſiderably from the truth; and therefore, to avoid tedious Calculations, our time will be better employed in ſearching for ſome ſimple Expreſſion by means of which we may in all Caſes determine the Reſiſtance of the Veſſel nearly. For this purpoſe let us ſuppoſe, as before, the greateſt Tranſverſe Section to be $= R$, and let the half Length of the Body or rather the Diſtance from the Fore Part of the Veſſel to this Section, or $AF = a$ and the half Breadth,

Plate I. Fig. 7.

$FM = b$ which is very near equal to the Depth: Now if the Fore Part of the Veſſel was a Parellopipedon, the Reſiſtance would be $= R$, and if it was Conical or a Pyramid terminating at A, the Reſiſtance would be as before

$$R.\frac{FM^2}{AM^2} = \frac{b^2}{a^2+b^2}.R.$$

Then ſince it is evident that the Figure of every Species of Veſſel now in uſe is included between theſe two Extremes, if we ſuppoſe that the actual Reſiſtance upon any given Figure is $= n.R$, the letter n will expreſs ſome Fraction between Unity and the Fraction $\frac{b^2}{a^2+b^2}$, we have therefore only to aſſign a juſt Mean between them. But having had recourſe to ſome Experiments made upon Ships of the Line, it appears that we ſhall approach ſufficiently near the Truth if we take $n = \frac{2b^2}{a^2+2b^2}$ the Harmonical Mean between 1 and $\frac{b^2}{a^2+b^2}$, we may therefore make uſe of this Expreſſion in all Caſes, at leaſt when the Fore Part of the Veſſel does not deviate conſiderably from the Forms commonly uſed in Ships of the Line, but ſhould this happen, it will not be difficult to determine to which of theſe two Limits the Reſiſtance will incline.

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§ 25.

Having therefore fixed upon $\frac{2b^2}{a^2+2b^2}$. R to expreſs the Reſiſtance of Veſſels; we obſerve that it only depends on the Proportion between the Length and Breadth of the Body, whoſe halves are denoted by the Letters *a* and *b*, whilſt the Letter R always repreſents the Reſiſtance of the greateſt tranſverſe Section of the Body moving through the Water in a Direction perpendicular to that Section: It will therefore not be amiſs to add the following Table expreſſing the Value of $\frac{2b^2}{a^2+2b^2}$. R, or rather the true Value of the Reſiſtance for each Proportion between the Length and Breadth of the Body.

Proportion $a : b$	Reſiſtance
$2 : 1$	$\frac{1}{3} \cdot R$
$2\frac{1}{2} : 1$	$\frac{8}{33} \cdot R$ or nearly $\frac{1}{4} R$.
$3 : 1$	$\frac{2}{11} \cdot R$
$3\frac{1}{2} : 1$	$\frac{8}{57} \cdot R$ or nearly $\frac{1}{7} R$
$4 : 1$	$\frac{1}{9} \cdot R$
$4\frac{1}{2} : 1$	$\frac{8}{89} \cdot R$ or nearly $\frac{1}{11} R$
$5 : 1$	$\frac{2}{27} \cdot R$
$5\frac{1}{2} : 1$	$\frac{8}{129} \cdot R$ or nearly $\frac{1}{16} R$
$6 : 1$	$\frac{1}{19} \cdot R$
$6\frac{1}{2} : 1$	$\frac{8}{177} \cdot R$ or nearly $\frac{1}{22} R$
$7 : 1$	$\frac{2}{51} \cdot R$

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Upon the Reſiſtance of Veſſels in their Oblique Courſes, and of Lee-way in general.

§ 26.

WHEN Veſſels are driven by the Wind, it is often impoſſible for them to follow a direct Courſe, and they are obliged to move in a Direction, which makes an Angle with the greater Axe; this Angle is called the Lee-way of the Veſſel. . We ſhall not here enter into a diſcuſſion of the Circumſtances that cauſe the Veſſel to move in an oblique Courſe, and therefore we only ſuppoſe it has ſuch a Motion, the Direction and Velocity of which are given; in this caſe we ſhall endeavour to determine the Reſiſtance which the Veſſel meets with from the Water. Now it is at firſt evident that the mean Direction of all the Efforts which the Water exerts upon the Surface of the Body, will no longer fall upon the diametrical Plane of the Veſſel, but it will be ſometimes on one ſide and ſometimes on the other; beſides, its Direction will not always continue parallel to the Horizon; indeed it is often impoſſible to reduce all the elementary Preſſures to a ſingle Force acting in a certain Direction;

however, it is always poſſible to reduce them to three Forces whoſe Directions are parallel to the three principal Axes of the Veſſel, viz. 1ſt, the horizontal Axe according to the Length of the Veſſel; 2dly, the horizontal Axe according to the Breadth; and 3dly, the Vertical Axe; all which interſect each other in the Center of Gravity of the Veſſel.

§ 27.

But as the conſideration of theſe three Forces in general would be very troubleſome, and beſides, not furniſh us with any uſeful Knowledge, we ſhall therefore begin our Reſearches by the moſt ſimple Caſe, which though inapplicable to Practice, will nevertheleſs furniſh us with juſt and clear Ideas upon the nature of the Subject which we are conſidering: Let us therefore ſuppoſe the Body to have the Form of a rectangular Parallelopipedon as expreſſed in the 10th Figure, where A B repreſents the greater Axe of the Body, C D its leſſer Axe, and F E the Vertical Axe, which at the ſame time marks the Depth of the Hollow: by this means all the Sections perpendicular to each of theſe three Axes, will be rectangular Parallelograms, and ſince the Planes which act againſt the Water, are Vertical, all the Efforts of the Reſiſtance will be in horizontal Directions, and there will

Plate II. Fig. 10.

not reſult any Vertical Force; beſides, ſince the Direction of Motion is always Horizontal, whatever Angle it makes with the Axe A B, all the horizontal Sections of this Body will meet with the ſame Reſiſtance from the Water: this will ſave the trouble of conſidering any other Section but that made at the Surface of the Water A B C D; therefore in all oblique Courſes, we have only to conſider the two ſides of the Parallelogram which are ſtruck by the Water, and having found the Reſiſtance which each of theſe Sides meet with, nothing more is required than to multiply them by the Depth of the Body F E, in order to have the whole Reſiſtance which the Body will experience from the Action of the Water.

§ 28.

Plate II. Fig. 11.

Let therefore the rectangular Parallelogram A B C D repreſent the horizontal Section at the Surface of the Water; alſo ſuppoſe A B its greater, and C D its leſſer Axe; and, for the ſake of brevity, let half the greater Axe or A F be denoted by a, and half the leſſer Axe or C F by b; ſuppoſe likewiſe that the Veſſel moves in the oblique Direction F X: then the Angle A F X will be the Lee-way of the Veſſel, which we ſhall repreſent by the Letter φ; now it is evident that the Face a A a $= 2b$ will be ſtruck by the Water under the

Angle A x F or a x X = 90° — φ whoſe Sine is = Coſ. φ, hence the Force of the Water will be expreſſed by 2 b. Coſ. φ^2, (but in effect it will be neceſſary to multiply this Expreſſion by the Depth of the Body, and alſo by $\frac{c^2}{4g}$; the Letter c denoting the Velocity of the Veſſel in the Direction F X, and this double multiplication muſt always be underſtood hereafter.) In the ſame manner, by drawing C c parallel to F X, we ſee that the Face a C b = 2 a, will be ſtruck by the Water under the Angle a C c = φ, and this Force will be = 2 a. Sin. φ^2. Now as both theſe Forces act directly upon, and paſs through the middle of the two Faces; therefore the firſt or 2 b. Coſ. φ^2 will act in the Direction A F, this we ſhall repreſent by the Line F r in the ſame manner as if it was applied at the Center F; and we ſhall alſo ſuppoſe the Line F s to be = 2 a. Sin. φ^2 which expreſſes the Force acting upon the Side a C b; then by compleating the Rectangle F r y s, the diagonal F y will repreſent the whole Force of the Reſiſtance which the Veſſel meets with in its Motion; this Force will therefore become

$$\text{F}y = \sqrt{4b^2.\,\text{Coſ.}\,\varphi^4 + 4a^2.\,\text{Sin.}\,\varphi^4},$$

and the obliquity of this Force with reſpect to the greater Axe A B will be the Angle B F y, whoſe Tangent is $\frac{ry}{Fr} = \frac{a.\ \text{Sin.}\ \varphi^2}{b.\ \text{Coſ.}\ \varphi^2}$.

§ 29.

Having determined this Force, it is evident that in order to continue the Motion of the Veſſel in the ſuppoſed Direction F X, it muſt be acted upon by a Force directly contrary to that of the Reſiſtance; therefore produce the Line y F towards Y, and F Y will give the Direction of the Force neceſſary to continue it in the Direction before mentioned. But we have juſt ſeen that the Tangent of the Angle A F Y is $= \frac{a\ \text{Sin.}\ \varphi^2}{b.\ \text{Coſ.}\ \varphi^2}$ from whence we already know the Relation between the Obliquity of the Courſe F X and that of the acting Force F Y, which is independent of the Velocity c of the Veſſel: but in order to have the Force itſelf which is neceſſary to preſerve the Veſſel in its Motion, we have only to multiply the Expreſſion juſt found $\sqrt{4\,a^2.\ \text{Sin.}\ \varphi^2 + 4\,b^2.\ \text{Coſ.}\ \varphi^2}$ by the Depth of the Body, and by $\frac{c^2}{4g}$; where it muſt be remembered that c denotes the Space through which the Veſſel runs in one Second, and g the Height which a heavy Body will fall in the ſame time by the Force of Gravity, this latter may be ſuppoſed equal to 16 Engliſh feet.

Beſides, we muſt not forget to expreſs this Force by the Weight of a Maſs of Water, equal in Volume to that which we have before aſſigned to it.

§ 30.

But that which here merits our attention moſt, is the Relation between the two Angles AFX and AFY, or rather between the Obliquity of the Courſe or Lee-way which is $= \varphi$, and the Obliquity AFY of the acting Force which we ſhall ſuppoſe $= \psi$, then the Tangent of ψ being $= \frac{a.\ \text{Sin}.\ \varphi^2}{b.\ \text{Coſ}.\ \varphi^2}$ we have $\frac{a}{b}.\ \text{Tang}.\ \varphi^2 = \text{Tang}.\ \psi$; hence knowing the Relation between a and b, it is eaſy to find for all the Angles φ thoſe of ψ which correſpond to them, and on the contrary for all the Angles ψ thoſe of φ may in like manner be found; for this laſt Caſe we have $\text{Tang}.\ \varphi = \sqrt{\frac{a}{b}.\ \text{Tang}.\ \psi}$: Since the Quantity a is in general much greater than b the Obliquity AFY of the acting Force will conſiderably exceed the Angle of the Lee-way AFX. However, it is evident from our Expreſſion that theſe two Obliquities will become equal when $\text{Tang}.\ \varphi = \frac{b}{a}$, for then we have alſo $\text{Tang}.\ \psi = \frac{b}{a}$, from whence it appears, that if the Lee-way was yet leſs, the other Obliquity ψ would be ſtill more

diminiſhed,

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diminiſhed, and whenever the Tang. $\varphi > \frac{b}{a}$, ψ becomes alſo $> \varphi$. In order to render this ſtill more evident, let α be an Angle whoſe Tangent is $= \frac{b}{a}$, then we ſhall have Tang. φ^2 $=$ Tang. α . Tang. ψ, or rather the three Angles α, φ and ψ have always ſuch a Relation, that their Tangents are in a geometrical Proportion, or the Tangent of φ is a Mean Proportional between the Tang. α and Tang. ψ.

§ 31.

Having already obſerved that the two Obliquities φ and ψ become equal when $\varphi = \alpha$, therefore ψ is alſo $= \alpha$; and it is evident that ſuch an Equality ought alſo to take place both when $\varphi = 0$ and when $\varphi = 90°$; for in the firſt Caſe the Courſe of the Veſſel becomes direct; and in the other, it moves in the Direction of the leſſer Axe F C, which Courſe we may alſo conſider as direct. Since therefore in theſe three Caſes, 1ſt, when $\varphi = 0$, 2dly, when $\varphi = \alpha$, and 3dly, when $\varphi = 90°$, we have $\psi = \varphi$, and that in every other Caſe theſe two Angles φ and ψ differ from each other; it will therefore be neceſſary to enquire in what caſe the Difference of theſe two Angles will become the greateſt, or rather when the Angle X F Y is the greateſt; by purſuing the common methods, we find that this Caſe will happen when the Sine of twice the Angle ψ is equal to half the Sine of twice the Angle φ, that is when Sin. $2\,\psi = \frac{1}{2}$ Sin. $2\,\varphi$, but the unraveling of this Queſtion depends upon the Reſolution of the following Equation of the 4th degree, viz.

$$\left.\begin{array}{l}\text{Tang. }\varphi^4 - 2\text{ Tang. }\alpha.\text{ Tang. }\varphi^3\\ -\,2\text{ Tang. }\alpha.\text{ Tang. }\varphi + \text{Tang. }\alpha^2\end{array}\right\} = 0,$$

the Roots of which can only be aſſigned by Approximation, ſo that this Queſtion which at firſt does not appear difficult, requires for each Value of α, Calculations that are very embarraſſing.

§ 32.

But as all that we have yet done is drawn from a Caſe which does not exiſt in Practice, it may be thought that we have employed too much time upon it; however, it will very ſoon be ſeen that the conſideration thereof may lead to ſome general Concluſions, which are applicable to almoſt all Veſſels; we have only to connect with the Letters a and b which expreſs the two Semi-Axes of our Figure, other notions that are equally conſonant. For if the Parallelogram ſhould move in the Direction of the greater Axe B A, the Reſiſtance will be $= 2\,b$; and if it ſhould move in the Direction of the leſſer Axe C D, the Reſiſtance will become $= 2\,a$; we may therefore conclude that our Expreſſions would become applicable to all Veſſels, provided that inſtead of 2 *b* we ſhould ſubſtitute the Reſiſtance which they meet with in their direct Courſes; and in place of 2 *a*, that which they would experience, if moved with the ſame Velocity in the Direction of the leſſer Axe: the former Reſiſtance we ſhall expreſs by the Letter P, and the latter by the Letter Q: Thus for any Veſſel whatever, we have only to write the Reſiſtances P and Q, inſtead of the Letters 2 *b* and 2 *a*, and then the Relation between the two Obliquities φ and ψ, which in the former Caſe was expreſſed by $\text{Tang.}\,\psi = \frac{a}{b}\,.\,\text{Tang.}\,\varphi^2$, will be generally expreſſed by $\text{Tang.}\,\psi = \frac{Q}{P}\,.\,\text{Tang.}\,\varphi^2$. For upon the leaſt reflexion, we perceive that the Letters *a* and *b* enter no otherwiſe into our Formula than as Expreſſions for the two Reſiſtances under our conſideration; from whence it follows, that the Force itſelf in the Direction F Y, requiſite to maintain the Veſſel in its Courſe F X will be $= \sqrt{P^2\,.\,\text{Sin.}\,\varphi^4 + Q^2\,.\,\text{Coſ.}\,\varphi^4}$: Theſe Expreſſions will ſeldom deviate from the truth, or at moſt their deviation will be very inconſiderable, and this conſideration leads us to the Subject of the following Chapter.

CHAP.

C H A P. V.

Upon the Relation between the Obliquity of the Courſe of a Veſſel, and that of its acting Force.

§ 33.

Plate III, Fig. 12.

AFTER our Reſearches upon the Relation between the two Angles φ and ψ, let us conſider any Veſſel whatever, whoſe three principal Axes of the Body are the greater Axe A B $= a$, the leſſer Axe C D $= b$, and the Vertical one or Depth F E $= e$, and let us ſee in what manner the Relation now under our conſideration, may be expreſſed by theſe three Dimenſions of the Body a, b and e. We ſhall at firſt, from the Principles already eſtabliſhed, determine the Reſiſtance which the Veſſel will experience in its direct Courſe in the Direction B A: for this purpoſe regard muſt be had to its greateſt tranſverſe Section C E D, whoſe Baſe C D being $= b$, and Height F E $= e$, its Area will be contained between the Limits $b\,e$ and $\frac{1}{2}\,b\,e$, which we ſhall therefore ſuppoſe $= \frac{3}{4}\,b\,e$; and we ſhall very ſoon ſee that a ſmall error in the Aſſumption of this Mean can be of little or no conſequence. Now this Area $\frac{3}{4}\,b\,e$

will expreſs the Reſiſtance which the Veſſel receives from the Water in a direct Manner, and which muſt always be underſtood as multiplied by $\frac{c^2}{4g}$; this will therefore expreſs the Value of the Letter R employed in the 3d Chapter, for determining the Reſiſtance in a direct Courſe: Hence, ſince the Ratio of *a* to *b* is the ſame here as we have there expreſſed by theſe Letters, it follows that the Reſiſtance of the Veſſel in its direct Courſe will be $= \frac{2b^2}{a^2 + 2b^2} \cdot \frac{3}{4} be$; and this will conſequently be the ſame Quantity which we have expreſſed by the Letter P at the end of the preceding Chapter; wherefore

$$P = \frac{2b^2}{a^2 + 2b^2} \cdot \frac{3}{4} be.$$

§ 34.

Let us now ſuppoſe the ſame Veſſel to move through the Water with the ſame Velocity *c* in the Direction of its leſſer Axe DC, and we ſhall immediately conceive that its Reſiſtance muſt be enormous, and to find it, we have only to conſider the Diametral Section of the Body AEB as ſtriking directly againſt the Water: Now the Area of this Section being equally comprized between the Limits *a e* and $\frac{1}{2}ae$. we ſhall therefore ſuppoſe it $= \frac{3}{2}ae$ which will alſo as before expreſs the Reſiſtance of this Section, and it is evident that the Curvature of the Veſſel cannot ſenſibly diminiſh this Reſiſtance; but in effect, if we here conſider b as the greater and a as the leſſer Axe, our Rule will in this Caſe give the Reſiſtance $= \frac{2a^2}{2a^2 + b^2} \cdot \frac{3}{4}ae$, and which is the ſame which we have before expreſſed by the Letter Q: From whence we deduce the Fraction

$$\frac{Q}{P} = \frac{a^3}{b^3} \cdot \frac{a^2 + 2b^2}{2a^2 + b^2}.$$

In this Equation neither the Depth e nor the Coefficient $\frac{3}{4}$ are concerned. Hence if a is ſeveral times greater than b, and therefore a^2 much greater than b^2, this Fraction will be nearly equal to $\frac{a^3}{2b^3}$, and as we have no great reaſon to expect a higher degree of exactneſs, we may boldly make uſe of this laſt Expreſſion.

§ 35.

It will now be very eaſy to attain the end we have in view; for which purpoſe, let us ſuppoſe the Veſſel to move Obliquely in the Direction FX, and that to maintain it in this Courſe, it muſt be acted upon according to the Direction

FY; if therefore we alſo ſuppoſe the Lee-way of the Veſſel AFX $= \varphi$, and the Obliquity of the acting Force FY, or the Angle AFY $= \psi$, we ſhall then have the Relation of theſe two Angles expreſſed by the Equation.

$$\text{Tang. } \psi = \frac{a^3}{2 b^3} \text{. Tang. } \varphi^2,$$

from whence we may eaſily determine the Value of ψ, when the other Angle φ is given: But if the Angle ψ was given, then φ would be found by ſolving the Equation

$$\text{Tang. } \varphi = \sqrt{\frac{2 b^3}{a^3} \text{. Tang. } \psi}.$$

It will not be even difficult to calculate Tables for each Species of Veſſel; and a ſmall number of ſuch Tables would be ſufficient, ſince the Proportion between the Length and Breadth of almoſt all Veſſels are included between the Limits 3 to 1 and 6 to 1, by which means the following Caſes expreſſing the Ratio of *a* to *b*, viz. 3 to 1; $3\frac{1}{2}$ to 1; 4 to 1; $4\frac{1}{2}$ to 1; 5 to 1; $5\frac{1}{2}$ to 1 and 6 to 1, will furniſh us with all the knowledge that we can deſire. Laſtly, ſince the Leeway or the Angle φ ſeldom exceeds 30 Degrees, it will be ſufficient to calculate theſe Tables for every 5 Degrees, and therefore they may be reduced to the following ones.

I Species,
where AB = 3 C D.

AFX	AFY
5°	5°: 54′
10	22 : 46
15	44 : 16
20	60 : 47
25	71 : 11
30	77 : 28
35	81 : 24

II Species,
where AB = $3\frac{1}{2}$ CD

AFX	AFY
5°	9°: 19′
10	33 : 41
15	56 : 59
20	70 : 36
25	77 : 54
30	82 : 2
35	84 : 34

III Species,
where AB = 4 CD

AFX	AFY
5°	13°: 46′
10	44 : 51
15	66 : 29
20	76 : 44
25	81 : 49
30	84 : 39
35	86 : 21

IV Species,
where AB = $4\frac{1}{2}$ CD

AFX	AFY
5°	19°: 14′
10	54 : 47
15	78 : 1
20	80 : 36
25	84 : 14
30	86 : 14
35	87 : 27

V Species,
where AB = 5 CD

AFX	AFY
5°	25°: 34′
10	62 : 46
15	77 : 26
20	83 : 7
25	85 : 47
30	87 : 15
35	88 : 8

VI Species,
where AB = $5\frac{1}{2}$CD

AFX	AFY
5°	32°: 29′
10	68 : 51
15	80 : 30
20	84 : 49
25	86 : 50
30	87 : 56
35	88 : 36

VII Species,
where AB = 6 CD

AFX	AFY
5°	39°: 35′
10	73 : 25
15	82 : 39
20	86 : 1
25	87 : 34
30	88 : 25
35	88 : 55

§ 36.

Although theſe Tables are principally intended to ſhew the Obliquity of the acting Force AFY for each Species of Veſſel, when the Leeway or Angle AFX is given, yet they may reciprocally be uſed to determine the Lee-way AFX when the Obliquity AFY is known. For example, in the 5th Species where $AB = 5\,CD$, if the Obliquity of the acting Force was $AFY = 62°$; we ſee that the Lee-way or the Angle AFX would then be nearly $= 10°$; but ſince the Angles AFY vary by ſuch great Intervals, it would be difficult to make uſe of a proper Interpolation, therefore ſince this Queſtion happens ſo frequently in Practice, it will yet be neceſſary to calculate another Table, in order to ſhew the true Value of the Lee-way AFX for each Species of Veſſel, and for every Angle AFY. For this purpoſe it will be ſufficient to conſider the Angles AFY for every 10 Degrees as far as 60°; from thence for every 5 Degrees as far as 80°, and from 80° to 85° for every ſingle Degree; and it would be uſeleſs to carry them beyond 85°. In order to calculate this Table we muſt make uſe of this Equation

$$\text{Tang. } \varphi = \sqrt{\frac{2b^3}{a^3}}.\ \text{Tang. } \psi\text{; whence Log.}$$

$$\text{Tang. } \varphi = \frac{1}{2}\ \text{Log. Tang. } \psi - \frac{1}{2}\ \text{Log. } \frac{a^3}{2b^3},$$

we ſhall therefore exhibit this Table in the following Manner.

A TABLE *ſhewing the Lee-way of each Species of Veſſel, the Obliquity of the acting Force* AFY *being given.*

Angle AFY	Length AB of the Veſſel.			
	3 CD	3½ CD	4 CD	4½ CD
10°	6°: 31′	5°: 11′	4°: 14′	3°: 33′
20	9 : 19	7 : 25	6 : 5	5 : 7
30	11 : 41	9 : 19	7 : 39	6 : 25
40	14 : 1	11 : 11	9 : 12	7 : 44
50	16 : 33	13 : 16	10 : 55	9 : 11
60	19 : 43	15 : 52	13 : 6	11 : 3
65	21 : 43	17 : 31	14 : 29	12 : 15
70	24 : 17	19 : 42	16 : 20	13 : 48
75	27 : 44	22 : 39	18 : 51	15 : 56
80	32 : 57	27 : 13	22 : 50	19 : 26
81	34 : 22	28 : 29	23 : 57	20 : 25
82	35 : 59	29 : 57	25 : 15	21 : 34
83	37 : 49	31 : 38	26 : 45	22 : 54
84	40 : 1	33 : 40	28 : 37	24 : 33
85	42 : 37	36 : 9	30 : 52	26 : 36

A TABLE *ſhewing the Lee-way of each Species of Veſſel, the Obliquity of the acting Force* AFY *being given.*

Angle AFY	Length AB of the Veſſel. 5 CD	$5\frac{1}{2}$ CD	6 CD
10°	3°: 2′	2°: 38′	2°: 19′
20	4 : 22	3 : 47	3 : 19
30	5 : 29	4 : 45	4 : 11
40	6 : 37	5 : 44	5 : 3
50	7 : 52	6 : 49	5 : 59
60	9 : 27	8 : 13	7 : 13
65	10 : 31	9 : 7	8 : 1
70	11 : 50	10 : 18	9 : 4
75	13 : 44	11 : 57	10 : 32
80	16 : 46	14 : 38	12 : 55
81	17 : 38	15 : 24	13 : 36
82	18 : 39	16 : 18	14 : 24
83	19 : 50	17 : 22	15 : 21
84	21 : 19	18 : 41	16 : 32
85	23 : 9	20 : 20	18 : 1

§ 37.

Now by the help of this Table it will be eaſy to reſolve the Queſtion before mentioned, where it is required to aſſign for each Species of Veſſel the two Obliquities, or the Angles AFX, and A F Y, ſo that their Difference, or the Angle X F Y may be the greateſt. But the reſolution of this Problem is of the utmoſt importance in the Art of Navigation, in order to know how to make the beſt Uſe of every Wind, as we ſhall ſhew in the following Book, in a more particular manner; and in the mean time we ſhall ſtate here the two Angles A F X and A F Y for each Species of Veſſel, when the Difference is the greateſt, which will be ſeen in the Table ſubjoined.

A TABLE *ſhewing the Angle of the Lee-way* AFX, *and that of the acting Force* AFY, *for each Species of Veſſel, when their Difference* XFY *is the greateſt.*

Species of Veſſel.	Angle AFX	Angle AFY	Difference XFY
AB = 3 CD	29°: 30′	76°: 53′	47°: 23′
AB = 3½ CD	26 : 4	78 : 56	52 : 52
AB = 4 CD	23 : 45	80 : 6	56 : 21
AB = 4½ CD	20 : 0	80 : 36	60 : 36
AB = 5 CD	18 : 27	81 : 53	63 : 26
AB = 5½ CD	16 : 18	82 : 6	65 : 48
AB = 6 CD	15 : 4	82 : 50	67 : 46

§ 38.

Hitherto we have only conſidered the Direction of the acting Force, but the Principles that we made uſe of for that purpoſe will alſo lead us to the knowledge of the Force itſelf, which is requiſite to impreſs the Veſſel with the given Velocity *c*, for we have before found (ſee 32d §) this Expreſſion for the Force.

$$\sqrt{P^2.\ \text{Coſ.}\ \varphi^4 + Q^2.\ \text{Sin.}\ \varphi^4}.$$

and it is proper to remark, that we have there this Equality $\text{Tang.}\ \psi = \frac{Q}{P} \cdot \text{Tang.}\ \varphi^2$ from whence we find

$$P = \frac{Q.\text{Tang.}\ \varphi^2}{\text{Tang.}\ \psi} \text{ and } P.\ \text{Coſ.}\ \varphi^2 = \frac{Q.\text{Sin.}\ \varphi^2}{\text{Tang.}\ \psi}:$$

Therefore the Force will be expreſſed by $\frac{Q.\ \text{Sin.}\ \varphi^2}{\text{Sin.}\ \psi}$: But we have found (ſee § 34th.)

$$Q = \frac{3}{4}\, a\, e\, .\, \frac{2\, a_2}{2\, a^2 + b^2},$$

and ſince it is ſtill neceſſary to multiply our Expreſſion by $\frac{c^2}{4g}$, the true acting Force will be equal to

$$\frac{c^2}{4g} \cdot \frac{3}{4}\, a\, e \cdot \frac{2\, a^2}{2\, a^2 + b^2} \cdot \frac{\text{Sin.}\ \varphi^2}{\text{Sin.}\ \psi}.$$

And becauſe we may without any impropriety neglect b^2, the Force required will become ſimply

$$\frac{c^2}{4g} \cdot \frac{3}{4}\, a\, e \cdot \frac{\text{Sin.}\ \varphi^2}{\text{Sin.}\ \psi}.$$

expreſſed by an equal Volume of Water. From thence therefore reciprocally if the acting Force was given $= F$, we might readily determine the Velocity which the Veſſel will receive, by the aid of this Equality $\frac{c^2}{4g} = \frac{4F}{3ae} \cdot \frac{\text{Sin.}\ \psi}{\text{Sin.}\ \varphi^2}$.

and, by this means, we know how to eſtimate the greatneſs of the acting Force: But with reſpect to the Place of application of this Force, that will be the Subject of Enquiry in the following Chapter.

§ 39.

But before this Chapter is finiſhed we ſhall take notice of a very ſingular Paradox in the Expreſſion,

$$\sqrt{P^2 . \text{Coſ.}\, \varphi^4 + Q^2 . \text{Sin.}\, \varphi^4}.$$

which denotes the Force of the Reſiſtance, and which becomes $= P$ when $\varphi = 0$, and will be $= Q$ if $\varphi = 90°$. The firſt of theſe Caſes will happen when the Veſſel moves in the Direction of the great Axe B A; and the other when it moves in that of the leſſer Axe C D, from whence it ſeems that ſince Q is ſeveral times greater than P, the leaſt Reſiſtance ought to take place in the direct Courſe, where the Lee-way φ vaniſhes. But it is certain that the Reſiſtance will become ſtill leſs for a particular Angle of Lee-way, which is found by taking the Tang. $\varphi = \frac{P}{Q}$ and muſt in conſequence be very ſmall; and ſince we then have Sin. $\varphi = \frac{P}{\sqrt{P^2 + Q^2}}$

and

and Coſ. $\varphi = \frac{Q}{\sqrt{P^2+Q^2}}$ there will reſult

$$P^2.\ \text{Coſ.}\ \varphi^4 = \frac{P^2.\ Q^4}{\overline{P^2+Q^2}^{\,2}} \text{ and}$$

$$Q^2.\ \text{Sin.}\ \varphi^4 = \frac{Q^2.\ P^4}{\overline{P^2+Q^2}^{\,2}}, \text{ therefore}$$

$$P^2.\ \text{Coſ.}\ \varphi^4 + Q^2.\ \text{Sin.}\ \varphi^4 = \frac{P^2.\ Q^2}{P^2+Q^2},$$

the ſquare Root of which will give the Reſiſtance $= \frac{P.\ Q}{\sqrt{P^2+Q^2}}$; which is leſs than P, becauſe $\frac{Q}{\sqrt{P^2+Q^2}}$ is leſs than Unity. Here is therefore this great Paradox, that by giving to the Veſſel a ſmall Lee-way φ, ſuch that Tang. $\varphi = \frac{P}{Q}$, the Reſiſtance will be found leſs than in the direct Courſe; but it is preciſely the Caſe which we have already remarked, where the Angle X F Y vaniſhes, ſo that the Reſiſtance is here directly contrary to the Motion F X.

C H A P. VI.

Upon the Place of Application of the acting Force.

§ 40.

FROM the Manner in which we have began this Reſearch, by conſidering the Body as a rectangular Parallelogram, and the Force of the Reſiſtance as applied to the Center of the Body F; and alſo when we have afterwards generalized this Hypotheſis, the Concluſions which we have drawn from it regard only the Quantity of the Reſiſtance F *y* and its Obliquity, or the Angle *r* F *y* which the Direction makes with the great Axe of the Veſſel A B; and we ſhould be much deceived if we were deſirous of extending this Generalization unto the Place of Application. In order to render this obvious we have only to conſider the Body as having the Figure of a Rhombus A C B D, where A B repreſents the great Axe and C D the leſſer Axe, and ſuppoſe it to move in the Direction F X, but in ſuch a manner that the Lee-way A F X be leſs than the Angle A B C; for on the one hand we have ſeen that the Angles of the Lee-way never become very conſiderable; and, on the other, it is ſufficient for our purpoſe to find the true Place

Plate II. Fig. 11.

Plate III. Fig. 13.

Place of the Application for thoſe which are ſmall. This being obſerved, it is evident that the Veſſel can only ſtrike the Water by its two Faces A C and A D; and ſince upon each the Obliquity of the Incidence is every where the ſame, the Mean Direction of the Force of the Water will paſs through the Points M and N, where theſe two Sides are divided into two equal Parts; and as theſe Forces are perpendicular to the Faces, let the Perpendiculars M Q and N Q be drawn, which will interſect the great Axe at Q; and therefore the Mean Direction of theſe two Forces, which is that of the Reſiſtance itſelf, will undoubtedly paſs through this Point Q.

§ 41.

In the preſent Caſe therefore it is evident that the Direction of the acting Force Q Y muſt paſs through the Point Q, or rather that the Force requiſite to maintain the Veſſel in its oblique Courſe, muſt be applied to the Point Q, or, more properly ſpeaking, to ſome other Point elevated directly above Q: But here we ſhall confine ourſelves to find the Interval between the Middle of the Veſſel F and the Point Q where this Force muſt be applied, without embarraſſing ourſelves at preſent with any Vertical Heighth, as that depends on particular circumſtances which will be explained hereafter.

ter. Now in order to determine the Point Q, draw C K perpendicular to the Side A C; then it is evident this Point muſt be ſituated at the middle of the Interval A K: then, becauſe A F : F C : : F C : F K, we ſhall have

$$FK = \frac{FC^2}{AF}, \text{ and therefore}$$

$$AK = AF + \frac{FC^2}{AF}; \text{ hence the Interval}$$

$AQ = \frac{1}{2}AF + \frac{FC^2}{2AF}$. It is therefore before the Center F of the Body towards A that the acting Force muſt be applied; and by ſuppoſing the whole Length $AB = a$, and the whole Breadth $CD = b$, we ſhall ſave the Interval $FQ = \frac{1}{4}a - \frac{b^2}{4a}$.

§ 42.

Let us now compare together the two Figures which we have given to the Body: the firſt, of a rectangular Parallelogram, has furniſhed us with the Interval $FQ = 0$; whilſt the preſent one, of a Rhombus, gives the Interval $FQ = \frac{a}{4} - \frac{b^2}{4a}$; from whence we conclude, that ſince all Figures of Bodies are equally comprized between theſe two Extreams, we may regard them as Limits for all Veſſels in general, and that the Interval F Q will hold a certain Mean between theſe two Values 0 and $\frac{a}{4} - \frac{b^2}{4a}$: Hence we draw this Concluſion, that by eſtabliſhing in general this Interval $FQ = \frac{a}{8} - \frac{b^2}{8a}$, no great error can poſſibly ariſe, and although it may happen ſometimes to be rather too great or too ſmall, yet this Difference will be almoſt always inſenſible and may well be neglected in Practice.

§ 43.

Having thus determined the Interval F Q, and ſince it is near the Point Q that the acting Force muſt be applied in the oblique Courſes of a Veſſel, we already perceive that it is in this Point Q where the principal Maſt of the Veſſel, or rather that which might be regarded as equivalent to all the Maſts taken together, muſt be eſtabliſhed; from whence the juſt Determination of this Point muſt be ſo much the more important. It is therefore neceſſary that the principal Maſt ſhould be placed nearer to the Head than the Stern of the Veſſel, and Builders are very nice upon the true Place for this Point Q, or upon the Proportion between the Diſtances A Q and B Q, which they in general eſtabliſh nearly as 2 to 3, this agrees ſufficiently well with our Determinations; for having

$AQ = \frac{3}{8}a + \frac{b^2}{8a}$ and $BQ = \frac{5}{8}a - \frac{b^2}{8a}$, the Proportion between A Q and B Q will become $3 + \frac{b^2}{a^2}$ to $5 - \frac{b^2}{a^2}$, which would agree with that of 2 to 3 if $\frac{b^2}{a^2}$ was $= \frac{1}{5}$. But it is alſo neceſſary to conſider that our Suppoſition, by which we have aſſigned to the Interval P Q, the Arithmetical Mean between the two Limits, might well deviate ſomething from the Truth. It appears even probable that the real Figures approach nearer to a Rectangle than to a Rhombus; and if we ſuppoſe $FQ = \frac{2}{5}.\overline{\frac{a}{4} - \frac{b^2}{4a}}$, we ſhall find

$$AQ = \frac{2}{5}a + \frac{b^2}{10a}, \text{ and}$$

$$BQ = \frac{3}{5}a - \frac{b^2}{10a}.$$

Therefore, by neglecting the very ſmall Term $\frac{1}{10}.\frac{b^2}{a^2}$, the Proportion will be truly as 2 to 3, which notable Harmony is both ſufficient to elucidate and confirm the Method that we have purſued.

§ 44.

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§ 44.

Let us now ſee what would happen, if we ſhould want to apply the acting Force in its true place, and at firſt, with reſpect to the progreſſive Motion of a Veſſel, it is certain that the Reſiſtance would always produce the ſame Effect in whatever Place the Force ſhould be applied, ſuppoſing the Direction to continue the ſame, and therefore a contrary Force would deſtroy its Effect in whatever Place it was applied; ſo that, with reſpect to the progreſſive Motion, the place of Application is abſolutely indifferent: But this is not the caſe with reſpect to the Inclination of a Veſſel, which depends on the Momentum of the Force of Reſiſtance, with reſpect to ſome Horizontal Axe paſſing through the Center of Gravity of the Veſſel, and therefore, although the contrary Force was equal to the Reſiſtance, it might well happen that the Inclination cauſed by the Reſiſtance, would not be deſtroyed by it, or that a new Inclination might reſult from it; but this commonly happens in all Oblique Courſes, and it then ſeems almoſt impoſſible to prevent the Veſſel from ſuffering a very ſenſible Inclination; from whence we readily comprehend that a Veſſel inclined muſt ſuffer another Reſiſtance beſides that which we have already aſſigned, and it appears in general, that the direct Reſiſtance

P muſt receive ſome Increaſe therefrom, whilſt that of the lateral one Q is a little diminiſhed, and therefore the Fraction $\frac{P}{Q}$ in the Expreſſion

$$\text{Tang.}\,\psi = \frac{P}{Q}\,.\,\text{Tang.}\,\varphi^2$$

muſt be ſomething augmented. However, this Expreſſion itſelf and the Tables which we have deduced therefrom, may nevertheleſs be ſufficiently exact, provided the Proportion between the Length and Breadth of the Veſſel be a little diminiſhed; thus for Example, if the Veſſel ſhould belong to the fourth Species, we might make uſe of the Tables appertaining to the third.

§ 45.

But the principal Object here, is the Momentum of the Reſiſtance with reſpect to the Vertical Axe of the Veſſel paſſing through its Center of Gravity: thus in Fig. 13th, where the Line Qy repreſents the Force of the Reſiſtance, and where the Vertical Axe paſſes through the Point F, the Momentum of this Force will be $= Qy.\ QF.\ \text{Sin.}\ FQy$, by which the Veſſel will be urged round the Vertical Axe in the Direction A *a*; therefore if the acting Force is not applied in ſuch a Manner as to be equivalent to it, the Veſſel will receive a rotatory Motion round its Vertical Axe, which

Plate III. Fig. 13.

which it is abſolutely neceſſary to deſtroy in order to preſerve the Veſſel in its Courſe; for if the Difference in the Place of Application was very conſiderable, the Action of the Rudder only would not be ſufficient to counterbalance this Effect. Hence we are ſufficiently apprized how neceſſary it is to obſerve the Rule which we have juſt found for the Place of Application of the acting Force, at leaſt nearly; ſeeing that the Effect which would reſult from a ſmall Deviation, might eaſily be deſtroyed by the Action of the Rudder; beſides, Navigators ought always to have ſome Sails at their Command, in order to aid the Action of the Rudder.

§ 46.

We have hitherto conſidered all the Forces as applied at the Surface of the Water; but it is eaſy to comprehend that the Height at which we apply the acting Force, muſt be principally taken into conſideration, when the Inclination which the Veſſel muſt ſuffer, both from the Reſiſtance and moving Force is required; ſince it is evident that the more this Place of Application is elevated, the more alſo will the Veſſel be inclined; and as in the Oblique Courſe, the Direction of the acting Force F Y is nearly perpendicular to the great Axe A B of the Veſſel, there will reſult from it a very

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very conſiderable Momentum to incline the Veſſel round this Axe, the Effect of which is to be apprehended in proportion as the Stability with reſpect to this Axe is diminiſhed: From whence we ſee that in order to render Veſſels proper for Oblique Courſes, it is abſolutely neceſſary to increaſe their Stability with reſpect to the great Axe: But this Matter will be conſidered in a more particular Manner in the following Book.

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CHAP. VII.

Upon the Action of the Rudder in a direct Courſe.

§ 47.

Plate III. Fig. 14.

LET A C B D repreſent a horizontal Section of the Body, at any Depth whatever under the Water, and let A B repreſent the great Axe which coincides with the Direction of the Motion, the Velocity of which ſuppoſe = c, alſo ſuppoſe the Vertical Axe of the Veſſel to paſs through the Point F; then let B K repreſent the Rudder fixed at any Obliquity whatever meaſured by the Angle b B K which we ſhall expreſs by the Letter ζ: It is now required to find the Effect of the Rudder in this Situation to make the Veſſel turn about its Vertical Axe, ſeeing it is proved by Mechanicks that all the Motions of Rotation muſt conſtantly be applied to the Axe which paſſes through the Center of Gravity of the whole Maſs. Thus we ought firſt to enquire the Force which the Rudder ſuſtains in this Situation, and afterwards determine the Momentum of this Force with reſpect to the Vertical Axe F G, or rather to the Point F.

§ 48.

§ 48.

But whilſt the Veſſel moves in the Direction B A with a Velocity $= c$, the Rudder B K will ſuffer the ſame Effort as if the Water ſhould ſtrike it in a contrary Direction with the ſame Velocity; for neither this Stroke or its Direction is effected by the Size of the Veſſel; but we eaſily conceive that both the Weight of the Veſſel as well as its Figure may conſiderably alter, not only the Direction, but alſo the Velocity with which the Water will ſtrike upon the Rudder: At the beginning of our Reſearches upon this Subject, we ſhall not take any Notice of theſe Irregularities, and we ſhall ſuppoſe that the Water ſtrikes the Rudder B K in the Direction A B or I L with a Velocity $= c$; but when we have reſolved this Caſe, it will be no longer difficult to eſtimate the Deviations which may ariſe from the before-mentioned Irregularities.

§ 49.

Since the Rudder is a Plane, and that the Water impinges upon it every where with the ſame Obliquity $BLI = bBK = \zeta$, the mean Direction of the Efforts of the Water will paſs through the Center of Gravity of the Surface of the Rudder which is immerſed in the Water,

this

this Point we ſhall ſuppoſe at L, then if the Area of this Surface be put $= f^2$, we have ſeen before, that the Force of the Water is equal to the Weight of a Maſs of Water, whoſe Volume is found by multiplying the Area f^2 by the Square of the Sine of the Angle of Incidence or ζ^2, and alſo by $\frac{c^2}{4g}$; ſo that this Force will be $= \frac{c^2 f^2}{4g} . \text{Sin.}\ \zeta^2$, and this Force will be applied to the Point L in the Direction L b, perpendicular to the Plane of the Rudder: But in order to form a better judgment of the Effect of this Force, we ſhall reſolve it into two lateral ones, the firſt in the Direction L p parallel to A B, the ſecond in the Direction L q perpendicular to it. For this purpoſe we ſhall ſuppoſe the Interval B L $= l$, then becauſe the Angle b B L $= \zeta$, we ſhall have B q $= l . \text{Coſ.}\ \zeta$ and L q $= l . \text{Sin.}\ \zeta$, therefore ſince the Angle L b p $= \zeta$, the Reſolution of the Forces will give us the Force in the Direction L p $= \frac{c^2 f^2}{4g} . \text{Sin.}\ \zeta^3$, and that in the Direction L q $= \frac{c^2 f^2}{4g} . \text{Sin.}\ \zeta^2 . \text{Coſ.}\ \zeta$; of which the former L p is directly oppoſed to the Motion of the Veſſel; but the latter L q will urge the Veſſel ſideways in the ſame manner as if theſe two Forces were applied to the Center of Gravity of the Veſſel, and to which the Action of the Rudder with reſpect to the progreſſive Motion of the Veſſel muſt be reduced.

§ 50.

Moreover, when theſe two Forces L p, L q are ſituated either above or below the Center of Gravity, they furniſh Momentums to incline the Veſſel, the firſt L p round the leſſer or tranſverſe Axe, and the other round the great Axe of the Veſſel: But ſince the Center of Gravity G of the Veſſel is commonly ſituated above the Point L, if we ſuppoſe this Heighth F G $= h$, the Momentum of the firſt Force L p will be $= \frac{c^2 f^2 . h}{4 g}$. Sin. ζ^3 by which the Veſſel will be inclined towards the Head, which in conſequence will be more immerſed in the Water, and the Momentum of the other Force L q will be $= \frac{c^2 f^2 . h}{4 g}$. Sin. ζ^2 . Coſ. ζ by which the Veſſel will be inclined towards the Starboard Side, or rather the Veſſel will be inclined towards the Side next the Rudder; from whence wee ſee that theſe Effects would be diminiſhed in proportion as the Center of Gravity G is ſituated lower in the Veſſel, and as the Heighth F G $= h$ can never become conſiderable, we eaſily conceive that nothing is to be apprehended from this Effect of the Rudder; and

CHAP. VII.

for this Reaſon we do not in common take it into conſideration.

§ 51.

But the principal Effect which the Rudder ought to produce, is relative to its Motion about the Vertical Axe G F, the knowledge of which we ſhall acquire from the Momentum of the Forces reſpecting the Vertical Axe G F itſelf: For the Force in the Direction L p being multiplied by the Interval L $q = l$ Sin. ζ, will give for the Axe F G, a Momentum equal to

$$\frac{c^2 f^2}{4g} . l . \text{Sin.}\ \zeta^4$$

by which the Head A will be turned towards the Starboard Side; and the other Force in the Direction L q being multiplied by the Interval q F = B q + B F will give a Momentum expreſſed by theſe two Terms,

$$\frac{c^2 f^2 . l}{4g} . \text{Sin.}\ \zeta^2 . \text{Coſ.}\ \zeta^2 + \frac{c^2 f^2}{4g} . \text{Sin.}\ \zeta^2 . \text{Coſ.}\ \zeta . \text{B F}$$

by which the Head will be alſo inclined to the Starboard Side, or towards the Rudder. Let us therefore add theſe two Momentums together, in order to have the whole Momentum which will cauſe the Veſſel to turn about the Axe F G in the Direction A a, this will conſequently be equal to

$$\frac{c^2 f^2 . l}{4g} . \text{Sin.}\ \zeta^2 + \frac{c^2 f^2}{4g} . \text{Sin.}\ \zeta^2 . \text{Coſ.}\ \zeta . \text{B F},$$

from whence we ſee at firſt, that if the Angle

b B K

bBK was $= 0$, or that the Rudder was in its natural Situation, then this Momentum of Force would entirely vaniſh; but if we made the Angle $\zeta = 90°$, this ſame Momentum would become $= \frac{c^2 f^2 . l}{4g}$, and therefore very inconſiderable, ſince the Line BF which is many times greater than the Interval l will in this Caſe entirely vaniſh out of the Calculation.

§ 52.

Therefore ſince the Rudder does not produce any Effect in the Caſe where $\zeta = 0$, and a very ſmall one when $\zeta = 90°$, it is evident that there will be a certain Mean Angle where the Rudder will produce the greateſt Effect poſſible: But to find this Obliquity which is the moſt Efficacious, we ſhall at firſt neglect in our Expreſſion the Term $\frac{c^2 f^2 . l}{4g} . \text{Sin.}\, \zeta^2$ which is very inconſiderable with reſpect to the other, ſo that we have now this Queſtion to reſolve; What Value muſt be given to the Angle ζ, in order that the Expreſſion $\frac{c^2 f^2}{4g} . \text{Sin.}\, \zeta^2 . \text{Coſ.}\, \zeta . \text{BF}$ or ſimply $\text{Sin.}\, \zeta^2 . \text{Coſ.}\, \zeta$ may become the greateſt poſſible: By the common Rules we find that this will happen when $\text{Tang.}\, \zeta = \sqrt{2}$, or rather when the $\text{Sin.}\, \zeta = \sqrt{\frac{2}{3}}$

and the Coſ. $\zeta = \sqrt{\frac{1}{3}}$; conſequently the Angle of this Obliquity b B K muſt be $= 54^\circ : 44'$, ſo that under this Angle the Rudder would produce the greateſt Effect to turn the Veſſel; and therefore our Momentum by neglecting the ſmall Interval l will be $= \frac{1}{6\sqrt{3}} \cdot \frac{c^2 f^2}{g} \cdot \mathrm{B\,F}$, but if we are deſirous to introduce the ſmall Interval B L $= l$ into our Calculation, by putting the Diſtance B F $= a$ we ſhall have this Expreſſion

$$\frac{c^2 f^2 . l}{4g} . \mathrm{Sin.}\, \zeta^2 + \frac{c^2 f^2 . a}{4g} . \mathrm{Sin.}\, \zeta^2 . \mathrm{Coſ.}\, \zeta, \text{ or}$$

$$\frac{c^2 f^2}{4g} \times \overline{l . \mathrm{Sin.}\, \zeta^2 + a . \mathrm{Sin.}\, \zeta^2 . \mathrm{Coſ.}\, \zeta},$$

which muſt be a Maximum, or ſimply

$$l . \mathrm{Sin.}\, \zeta^2 + a . \mathrm{Sin.}\, \zeta^2 . \mathrm{Coſ.}\, \zeta;$$

here by the uſual Methods we find this Equality

$$3\, \mathrm{Coſ.}\, \zeta^2 + \frac{2l}{a} . \mathrm{Coſ.}\, \zeta - 1 = 0.$$

But ſince $\frac{l}{a}$ is always a ſmall Fraction, we ſee evidently that the Angle ζ can differ but little from the preceding one, which is $= 54^\circ\, 44'$, hence we draw this ſimple Approximation; to add to the Angle $54^\circ : 44'$ as many Degrees as the Expreſſion $\frac{23 . l}{a}$ contains Units, or rather we ſhall have $\zeta = 54^\circ : 44' + \frac{23 . l^\circ}{a}$:

§ 53.

But this Determination is only true when the Water can run freely upon the Rudder in the Direction A B or I L, which never will happen but at the loweſt horizontal Section of the Body where it is terminated by the Keel, becauſe this Section being nearly a ſtreight Line, does not hinder the Water from running in the Direction I L with its entire Velocity $= c$; but whenever any horizontal Section of the Body is made above the Keel, we ſhall then have towards the Middle a very conſiderable Breadth, and this will prevent the Water from running freely upon the Rudder: We ſee plainly that if the Length of the Rudder B K was much greater than the half Breadth of that Section, the Water might then run freely at leaſt upon its extremity K; but ſince this Length B K is in general much leſs than the half Breadth F D, the Motion of the Water upon the Rudder will be more effected the more it approaches the Point B; and for this Reaſon the Conſideration upon the Effect of the Rudder will become ſo much the more complicated: It is even neceſſary here to acknowledge, that the Theory of the Motion of Fluids is not ſufficiently aſcertained for us to diſcover how much the Direction and Velocity of a Fluid which paſſes near a ſolid Body will be altered by it; however we ſhall endeavour to throw as much Light upon this Queſtion, as may at the ſame time be both ſatisfactory and ſufficient for Practice.

§ 54.

Plate III. Fig. 15.

Suppoſe therefore A C B D to be the Figure of ſuch a Section, and let A B be the Length and C D the Breadth, alſo ſuppoſe the Velocity of the Veſſel in the Direction B A to be as heretofore $= c$ and B K the Poſition of the Rudder under any Obliquity $= \zeta$; likewiſe let L be the Center of Gravity of the Surface of the Rudder or rather of the Part which correſponds to this Section; This being done it is evident that the Water could not flow upon the Rudder near the Point B, but in the Direction c B which is that of the Side of this Section near the Stern, and we eaſily conceive that if this Section ſhould preſerve its half Breadth F C almoſt to the Stern, that there would then remain but a very inconſiderable Curvature for the Side c B, in which Caſe the Water near B would not have any Motion, and the Rudder would not produce any Effect. It is neceſſary therefore, that the Side A C B ſhould in no Part have any great Curvature and ſtill leſs any Angularity, and that the Breadth F C ſhould diminiſh little by little towards the Point B with as little Curvature as Circumſtances will admit of.

§ 56.

Let us now ſuppoſe that the Water actually flows near B in the Direction c B, and let us put the Angle C B $c = \epsilon$, then ſince the Length of the Rudder B C is always very ſmall in compariſon with the greatneſs of the Veſſel, we may well ſuppoſe that the Water runs in the ſame Direction upon the whole Rudder K B. For this purpoſe let the Line L i be drawn parallel to B c in order to repreſent the Direction of the Water, alſo draw the Line L I parallel to B A then ſince the Angle B L I is $= \zeta$, and becauſe I L $i = \epsilon$, the Obliquity under which the Rudder will be ſtruck, will be $= \epsilon + \zeta$; and with reſpect to the Velocity itſelf we ſhall very ſoon prove that it is no longer $= c$ but rather $= c \,.\, \text{Coſ.}\, \epsilon$, by which means the Expreſſion for the preceding Caſe may eaſily be applied to the preſent one, by ſubſtituting $c \,.\, \text{Coſ.}\, \epsilon$ in place of c, and the Angle $\epsilon + \zeta$ in place of ζ, from thence we ſhall have the Force with which the Rudder will be ſtruck by the Water (ſee § 49) expreſſed by

$$\frac{c^2 \,.\, \text{Coſ.}\, \epsilon^2}{4g} \,.\, f^2 \,.\, \text{Sin.}\, \overline{\epsilon + \zeta}^{\,2} \,.$$

Hence we ſee that the more the Angle ϵ

approaches 90° the leſs will this Force become, and it will vaniſh entirely if the Angle C B *c* ſhould become a right one. But this is preciſely the Caſe where we have already remarked, that the Water near the Stern would not produce any Effect upon the Rudder. And in order to find the Momentum of this Force with reſpect to the Vertical Axe of the Veſſel, we may well neglect the ſmall Term which before contained the Letter l; (ſee § 51), then the other which is the principal one, will be found by multiplying the Force both by the Interval B F and by the Coſine of the Obliquity of the Rudder, or rather by the Coſ. ζ; hence the Momentum of Force of the Rudder will in the preſent Caſe become

$$\frac{c^2 \,.\, \text{Coſ.}\, \beta^2}{4g} \,.\, f^2 \,.\, \text{Sin.}\, \overline{\beta + \zeta}^2 \,.\, \text{Coſ.}\, \zeta \,.\, BF,$$

§ 56.

Now in order to find the Angle ζ, or the Obliquity of the Rudder which would produce the greateſt Effect, the common Methods will furniſh us with this Rule; that we firſt find an Angle γ ſuch that the Coſ. $\gamma = \frac{1}{3}$ Coſ. β, and then take $\zeta = 90° - \frac{\beta + \gamma}{2}$. Thus if the Angle β or C B *c* was $= 45°$, or the Angle

$CBD = 90^\circ$, which is nearly the Caſe at the Level of the Water, we ſhall then have Coſ. $\beta = 7071068$, and therefore

$$\frac{1}{3}\,\text{Coſ.}\,\beta = 2357023 = \text{Coſ.}\,\gamma$$

from whence we find

$\gamma = 76^\circ : 22'$; hence $\frac{\beta + \gamma}{2} = 60^\circ : 41'$; and

conſequently the Angle $\zeta = 29^\circ : 19'$, which is therefore much leſs than that appertaining to the loweſt Section of the Body, where we find $\zeta = 54^\circ : 44'$ and this Value may likewiſe be determined from our Rule by taking $\beta = 0$; for then the Coſine of $\beta = 1{,}0000000$ and the Coſ. $\gamma = 0{,}3333333$ from thence

$\gamma = 70^\circ : 32'$; therefore $\frac{\beta + \gamma}{2} = 35^\circ : 16'$,

and conſequently $\zeta = 54^\circ : 44'$ the ſame as before. Hence we may draw a very important Concluſion for Navigators, that in order to obtain the moſt ſpeedy and greateſt Effect from the Action of the Rudder, it is neceſſary to give to it an Obliquity ſomething leſs than that of $54^\circ : 44'$ which has been hitherto preſcribed by Mathematicians: For if the higheſt or Water-Section of the Body requires an Obliquity of $29^\circ : 19'$ whilſt the loweſt Section would demand one of $54^\circ : 44'$, it is neceſſary without doubt to make choice of ſome Mean between theſe two Limits; the Arithmetical Mean

CHAP. VII. Mean would give an Angle $= 42^\circ : 1'$; but as the Rudder has a much greater Breadth at Bottom than at the Level of the Water, and beſides the Effect of the Water much greater below than at the Surface, the Mean to be fixed upon ought to be much nearer the greateſt Limit. From whence it ſeems that we might eſtabliſh this Rule, that an Obliquity of about 48° will in general produce the beſt Effect; or rather in order to obtain the greateſt Effect, the Helm ought to make an Angle of 48° with the Axe of the Veſſel or at leaſt one of 45°, ſince the differences which ariſe near a Maximum are almoſt inſenſible.

§ 57.

At preſent there only remains to explain the Reaſons which have determined us to fix the Velocity with which the Water ſtrikes the Rudder $= c$. Coſ. β. For this purpoſe let the Plate IV. Fig. 16. preſent Poſition of the Section of the Body be repreſented by the Figure A B M where A B is ſuppoſed to be the great Axe of the Veſſel, and B M any Part whatever of its Side, therefore the Angle A B M $= \beta$; and ſuppoſe alſo, that in one Second of Time this Figure is advanced to $a\ b\ m$ through the Space B b $=$ M $m = c$, the Space gone over in one Second being always repreſented by c. But in this State if the Water which invironed the Veſſel in the Poſition ABM could not follow the Veſſel in its Motion, the Space B b m M would remain void; but the State of Preſſion with which Water is endowed, obliges it moſt readily to follow the Veſſel and to fill the Space B b m M, this Succeſſion will even be made in the ſhorteſt Diſtance; therefore drawing the Perpendicular m N from the Point m upon the Side BM, it is then plain that the Water will move from N to fill the Void near m, and this Succeſſion being made through the Space N m in one Second, its Velocity will be expreſſed by this ſame Interval: therefore ſince the Interval $\mathrm{M}\,m = \mathrm{B}b = c$ and the Angle $m\,\mathrm{MN} = \mathrm{ABM} = \beta$, the true Velocity of the Water at N will be $= c\ .\ \mathrm{Sin.}\ \beta$ in the Direction N m. Now in order to find the Velocity with which the Water ſtrikes the Rudder, it is neceſſary to conſider the Veſſel as at Reſt and the Sea as running againſt the Veſſel with a Velocity $= c$ in the Direction A B; therefore a Particle of Water at N will beſides its proper Motion be tranſported to n, by drawing N n parallel and equal to b B; conſequently by combining this Motion through N n with its proper Motion through N m, we have only to compleat the Parallelogram M m N n, whoſe Diagonal will repreſent both the Direction as well as the Velocity with which the Water at N is moved with reſpect to the Veſſel. Therefore ſince M m = c and the Angle m M N = ϵ, this Diagonal N M is evidently = c. Coſ. ϵ, ſo that the Velocity with which the Water ſtrikes the Rudder is in effect = c. Coſ. ϵ, as we have before ſuppoſed it, and as the Direction N M is alſo the ſame as we have aſſigned it, this Theory would appear to be ſufficiently eſtabliſhed.

CHAP. VIII.

Upon the Action of the Rudder in Oblique Courſes.

§ 58.

Plate IV. Fig. 17.

WE ſhall begin here in the ſame manner as before, by conſidering the loweſt Horizontal Section at the Keel of the Veſſel. Let the right Line A B therefore repreſent to us the Keel moving in the Direction A a or in that of F X with a Velocity = c, ſo that the Angle A F X will expreſs the Leeway of the Veſſel, which we ſhall ſuppoſe as before = φ, and we ſhall alſo ſuppoſe that the Rudder B K makes with the Keel prolonged the Angle K B S = ζ, this Angle of Obliquity being to Leeward. It is required therefore to determine both the Velocity and Direction with which the Water will ſtrike againſt the Rudder B K: For this purpoſe we ſhall conſider the Veſſel as at Reſt, and that the Water runs upon it in the Direction *a* A or X F with a Velocity $= c$; it is at firſt evident, ſince the Body of the Keel is oppoſed to the Continuation of this Motion, that the Water in approaching it will be obliged to change its Direction by Degrees in ſuch a Manner that near the Stern Poſt at B it will exactly follow the Direction of the Keel F B, and that with a diminiſhed Velocity, which we might conclude to be $= c$. Coſ. φ. But at ſome Diſtance from the Keel the Direction of the Water will approach nearer to its natural Direction X F, and that in Proportion as the Diſtance from the Keel is increaſed; but as the Rudder has little Extent, if we conſider its middle Point to be at L, by drawing there the right Line I L parallel to the Keel, and repreſenting the Direction of the Water by the Line *i* L, the Angle I L *i* will always be leſs than the Leeway A F X $= \varphi$; for the ſame Reaſon the Velocity there will alſo be greater than c. Coſ. φ, and as we cannot determine any thing with Preciſion, we ſhall take ſome other Angle θ leſs than φ; let us ſuppoſe therefore, the Angle I L $i = \theta$, and the Velocity of the Water

$= c \,.\, \mathrm{Coſ.}\, \theta$; then the Force of the Water upon the Rudder will become

$$\frac{c^2 .\, \mathrm{Coſ.}\, \theta^2}{4g} .\, f^2 .\, \mathrm{Sin.}\, \overline{\zeta + \theta}^2 .$$

becauſe the Angle of Incidence $\mathrm{B L} i = \zeta + \theta$; where f^2 denotes the Surface of the Rudder at this Place; but the Line L S which is perpendicular to it will indicate the Direction of the Force.

§ 59.

From hence we perceive that in this Reſearch it is neceſſary to have recourſe to ſome means, by which we may eſtimate the Direction of the Water at the Point L: but as an exact Determination would not be of any particular Advantage in Practice, ſince it is ſufficient to know in groſs that the Water effectively ſtrikes the Rudder, and as the Momentum of this Force with reſpect to the Vertical Axe F G of the Veſſel, by neglecting the ſmall Portion which depends on the Interval B L, will as before (ſee § 55.) be equal to

$$\frac{c^2 .\, \mathrm{Coſ.}\, \theta^2}{4g} .\, f^2 .\, \mathrm{Sin.}\, \overline{\zeta + \theta}^2 \cdot \mathrm{Coſ.}\, \zeta \,.\, \mathrm{B F}.$$

Therefore we may already conclude, that in order to obtain the greateſt Effect the Angle ζ muſt be leſs than 54° : 44′, and if we are deſirous to know this Angle more exactly, we have only to find an Angle η ſuch that Coſ. $\eta = \frac{1}{3}$ Coſ. θ and then take the $\zeta = 90° - \frac{\eta + \theta}{2}$. Moreover ſince the Leeway φ, ſeldom ſurpaſſes 20 Degrees, by taking $\theta = \frac{1}{2}\varphi$, this Expreſſion will not deviate conſiderably from the truth; therefore ſince θ does not ſurpaſs 10 Degrees, we ſee clearly that there would not reſult any ſenſible Error whenever this Angle ſhould be taken ſome Degrees greater or leſſer. Thus, by taking $\theta = 10°$ we find $\eta = 70° : 50'$ and $\zeta = 49° : 35'$; but in this Hypotheſis itſelf the firſt Factor c^2 . Coſ. θ^2 will not be conſiderably diminiſhed by the Multiplication of Coſ. θ^2, ſo that it would be of very little Importance to pretend to a higher degree of Preciſion.

§ 60.

Plate IV. Fig. 18.

But if the Rudder was turned to Windward, the Caſe would be entirely different from the preceding one, ſince there would not arrive upon the Rudder any other Water but that from beyond the Fore Part A, in the Direction A α, which if it ſhould preſerve its Direction would certainly never ſtrike the Rudder, was it even longer than common. But we conceive eaſily that the Water, which has began to move in the Direction

CHAP. VIII. A α, will change its Courſe by Degrees, and it will be curved nearly according to the Line $\alpha\, \beta\, \gamma$, ſo that ſome Portion might reach the Extremity of the Rudder; however the Force which reſults from it will be always much leſs than in the preceding Caſe. We know alſo but too well from Experience, that in theſe Caſes it is almoſt impoſſible to make a Veſſel turn to Windward by means of the Rudder; and Navigators are commonly obliged to uſe ſome Sails in order to attain this End: neither does it appear how it would be poſſible to remedy this Inconvenience unleſs we ſhould eſtabliſh a Rudder at the Fore Part of the Veſſel, or at the Extremity of the Stem: But it ſeems that this Method would meet with other Obſtacles abſolutely unſurmountable.

§ 61.

Plate IV. Fig. 19. Let us now conſider a Section of the Body more elevated, whoſe Breadth is repreſented by the Line C D, the Length being deſcribed as heretofore by the Line B A; alſo let the Motion be made in the Direction F X, and ſuppoſe the Angle of the Leeway A F X $= \varphi$; then, ſince the Rudder B K is turned to Leeward, its Obliquity being repreſented by the Angle S B K $= \zeta$; it is now evident,

that in this Caſe, the Water will run more freely upon the Rudder than in a direct Courſe, and conſequently will not loſe ſo much of its Velocity, therefore the Determinations which we have found in the preceding Chapter will ſtill anſwer our Purpoſe: But as the Obliquity with which the Water ſtrikes the Rudder is at preſent alſo greater for the loweſt Section of the Body, it follows, that in order to produce the greateſt Effect, the Angle ζ muſt be taken ſtill leſs than before; and perhaps it will be very proper not to increaſe the Angle S B K beyond 40 Degrees. However this Remark is not of any great Conſequence in Practice, ſince there is no Difficulty in knowing whether a Veſſel obeys the Rudder or not, and what Obliquity will produce the beſt Effect.

§ 62.

But the greateſt Difficulty we meet with is when the Rudder B *k* is turned to Windward, in which Caſe we ſee evidently that the Water coming from the Head in the Direction A *a* might perhaps not arrive upon the Rudder, although its Directions were ſomething curved; and it is alſo in this Situation that Veſſels in general do not anſwer their Helm; but even if the Water ſhould arrive at the Rudder

B *k*, its Effect would be always much leſs than in the direct Courſe; and we have only to conſider the Figure in order to aſſure ourſelves, that the ſhorter the Veſſel is with reſpect to the Breadth, the more this Defect muſt be obſerved. But if on the contrary, the Length of the Veſſel ſhould be many times greater than its Breadth, and the Figure of the Aft Part be finely ſhaped towards the Rudder, ſo that the Water might eaſily glide along the Sides of the Veſſel, it may well happen that its Effect upon the Rudder would be ſtill conſiderable, which will be always a very good Property belonging to thi Species of Veſſels: We alſo ſee that Builders are accuſtomed to narrow the Figure of the Aft Part inſenſibly, and to avoid all Curvature as much as poſſible; this Practice is only followed to obtain the great End of commanding the Veſſel by means of the Rudder: They have alſo contrived another Method, very proper to remedy this Defect, which is to give the Keel a Poſition inclined to the Horizon; ſo that the Stern Part, and therefore alſo the Rudder, are both immerſed to a greater Depth than the Fore Part of the Veſſel: For it is evident the Water will by this means be leſs obſtructed in ſtriking the lower Part of the

Rudder: There happens ſtill in this Caſe a favourable Circumſtance which facilitates the Action of the Rudder, for when the Veſſel is upon a Wind, and the Leeway is for Example on the Starboard Side, the Veſſel will Heel very conſiderably to Leeward, from whence it follows that the Larboard Side of the Keel muſt be turned up, and therefore the Body of the Veſſel does not ſo much impede the Water from ſtriking the Rudder.

§ 63.

What has been here ſaid upon the greateſt Action of the Rudder, muſt not be underſtood as a Rule which we ought to follow at all times; for whilſt a Veſſel continues in its Courſe, the Uſe of the Rudder is not neceſſary, except when a ſmall Deviation has by any Accident whatever taken place, and it is required to regain the true Direction; ſuch an Effect may in general be produced by a very ſmall Motion of the Rudder, and it would be very improper in ſuch a Caſe, to give the Rudder an Obliquity which might produce the greateſt Effect poſſible; and it is only when we have occaſion to turn the Veſſel ſuddenly that we muſt have recourſe to the moſt Efficacious Action of the Rudder: But in what Manner, and by what Motion the Veſſel will

CHAP. VIII. will be actually turned by the Force of the Rudder, that will be examined in the following Chapter.

CHAP. IX.

Upon the Motion of Rotation in Veſſels, produced by the Action of the Rudder.

§ 64.

CHAP. IX. IN order to determine the Motion of Rotation, which a Veſſel will receive from the Action of the Rudder, it is neceſſary above all things to determine truly the Momentum of this Force, with reſpect to the Vertical Axe of the Veſſel. But we have juſt ſhewn, that this Momentum will always be comprized in an Expreſſion of this Form,

$$\frac{\alpha . c^2}{4g} . f^2 . BF.$$

where f^2 repreſents the Surface of the Rudder; BF the Interval between the Rudder and the Vertical Axe of the Veſſel; c the Velocity of the Veſſel; and α a numerical Coefficient which reſults both from the Obliquity of the Rudder, the Leeway of the Veſſel, and alſo from the

Figure of the Stern Part; ſo that this Expreſſion contains four linear Dimenſions, of which three give a Volume of Water whoſe Weight repreſents the Force, and this being multiplied by the fourth Line, gives that which we call the Momentum of Force: From hence we ſee that this Momentum is always proportional to the Square of the Velocity c^2, conſequently the more rapid the Motion of the Veſſel becomes, the greater will be the Effect produced from the Rudder, which is indeed ſelf-evident, ſeeing that a Veſſel at Reſt is always inſenſible to the Rudder. It is alſo equally evident that this Force muſt be proportional to the Surface of the Rudder f^2; and beſides, it is likewiſe proportional to the Interval B F: Therefore the greater this Interval is, or the Length of the Veſſel exceeds its Breadth, the greater alſo will be the Efficacy of the Action of the Rudder; by which it appears that long Veſſels, beſides the great Advantages which we have already remarked, would in Addition have that of anſwering the Helm more readily.

§ 65.

But the Knowledge of this Momentum is ſtill not ſufficient to determine the true Motion which will be communicated to the Veſſel, we yet require another Element ariſing from the Maſs of the Veſſel itſelf; in the ſame manner as if it was required from the Progreſſive Motion to have that which we call Acceleration; it would then be neceſſary to divide the Moving Force by the whole Maſs. But ſince the preſent Queſtion is upon the Motion of Rotation, it is neceſſary to divide the Momentum of Force by a Quantity which we call the Momentum of Inertia of the Body with reſpect to the Axe of Rotation: from the Rules of Mechanicks this Momentum of Inertia is found, by multiplying each ſeparate Maſs of which the Veſſel is compoſed, by the Square of its reſpective Diſtance from the Axe of Rotation: If we therefore multiply all theſe Weights, each by the Square of its Diſtance from the Vertical Axe F G, there will reſult a Product of the entire Weight of the Veſſel M, by the Square of a certain Diſtance, which will be ſome Mean between the greateſt and leaſt Diſtances, which ſuppoſe $= k$; our Momentum of Inertia will then be $= M . k^2$; or rather, if we are deſirous of reducing the Weight of the Veſſel to a Volume of Water, as we have before done in the Momentum of Force, it will be only neceſſary to ſubſtitute inſtead of M, the Volume of the immerſed Part or the Body, which we before expreſſed by the Letter V; in this Caſe the Momentum of Inertia will be

$= V$

$= V . k^2$. This Formula therefore contains five Linear Dimenſions.

§ 66.

Now in order to find the Acceleration in the Motion of Rotation, it is neceſſary, according to the Rules of Mechanicks, to multiply the Momentum of Force by 2 g, or by twice the Heighth which heavy Bodies fall in one Second of Time, and divide this Product by the Momentum of Inertia of the Veſſel; this Acceleration will then be expreſſed by

$$\frac{1}{2} \cdot \frac{\alpha c^2 f^2 . BF}{V . k^2}.$$

which having five Linear Dimenſions in both Numerator and Denominator will give a Numerical Fraction that will always expreſs the Sine of the Angular Velocity generated in one Second, but it is neceſſary to obſerve, that we meaſure an Angular Velocity by the Angle which it is capable of moving through in one Second: From hence we comprehend that the Angle itſelf which the Change of Situation will produce in the Veſſel in the firſt Second will be half the angular Velocity that we have juſt found. And with reſpect to the Motion afterwards, we know that the angular Velocities acquired would be proportional to the Times, and the Angles themſelves deſcribed by this Motion of

Rotation to that of the Squares of the Times elaſped, if the Veſſel did not meet with any Reſiſtance in this Motion, and the moving Force remained the ſame.

§ 67.

But whenever the Veſſel begins to turn round its Vertical Axe, both its Direction and Velocity will experience ſome Change, and it is evident that the Force of the Water upon the Rudder will be alſo changed by it, by this means the Motion of Rotation would be no longer determined by the ſame Momentum of Force: Beſides, as ſoon as the Veſſel turns round its Axe, it will meet with a peculiar Reſiſtance in the Water, which tends to diminiſh this Motion. However, as this Motion is ſtill very ſlow, neither the Change in the Force or the Reſiſtance would be ſenſible, and therefore for the ſmall time of one Second we may well regard the Motion generated as agreeing with our Expreſſion, ſo that after one Second the Velocity of Rotation will be nearly the ſame as that which we have juſt aſſigned to it. But here an abſolute Meaſure of this Motion is not ſo much required as the Proportion which in this reſpect is found in different Species of Veſſels: Thus, as ſoon as we know the Proportion between the Quantities f^2, B F, V, and k^2 for two different Veſſels, and their Velocities c under the ſame Circumſtances; we are then able to judge which of theſe Veſſels will beſt anſwer the Helm, and alſo to determine the Ratio of their Velocities of Rotation round their Vertical Axes.

§ 68.

In order that this Proportion may be fully explained, let us ſuppoſe as heretofore the Length of the Body $= a$, the Breadth $= b$, and the Depth $= e$, then the Volume of the Body will be nearly proportional to the Product abe, and with reſpect to the Square k^2 we readily conceive that it muſt depend both on the Length a, and the Breadth b, ſo that we ſhall not deviate much from the truth by ſuppoſing it proportional to the Product ab; the Size of the Rudder is commonly regulated by the Breadth of the Veſſel, and as the Depth e conſtitutes the principal Dimenſion, we may regard the Surface f^2 as proportional to the Product be; laſtly, the Interval B F is evidently proportional to the Length a: From thence it follows that the Velocity of Rotation produced in one Second, or in any other ſmall Interval of Time will be always proportional to this Expreſſion $\frac{\alpha c^2}{ab}$, where the Coefficient α contains the ſmall Differences cauſed by the Diverſity of the Conſtructions,

ſtructions, and of the Courſe itſelf: Therefore the Motion of Rotation is directly as the Square of the Velocity, and reciprocally as the Product *a b*, or rather as the Area of the Water Section. Thus if two Veſſels are perfectly alike, of which one has all its Dimenſions twice as great as the other, the Velocity of Rotation of the greateſt will be only one fourth Part of that which will be acquired by the leſſer, under the ſame Circumſtances.

§ 69.

Before this Book is finiſhed we ought ſtill to ſay ſomething upon the Force which muſt be employed to preſerve the Rudder in a given Obliquity; for this purpoſe let the Obliquity of the Rudder K B S $= \zeta$ to which the Obliquity of the Incidence of the Water upon the Rudder is nearly equal, then the Force with which it will be ſtruck, will be

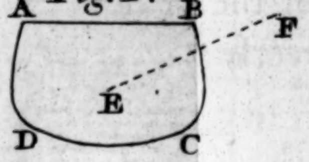
$$\frac{c^2 . f^2}{4g} . \text{Sin. } \zeta^2$$

this being multiplied by the Interval B L $= l$, the Point L being the Center of the Rudder, we ſhall have the Momentum of Force with reſpect to the Axe B, round which the Rudder itſelf is moved. Therefore ſuch a Force muſt be applied to the Helm as muſt produce a Momentum $= \frac{c^2 . f^2 . l}{4g} . \text{Sin. } \zeta^2 .$ from whence we ſee

that

Plate I. Book 2.

Fig. 1.



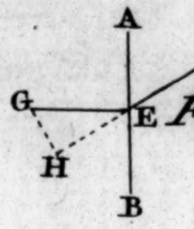
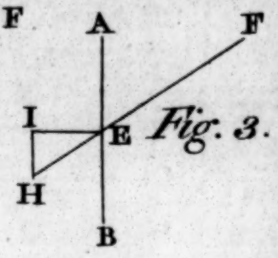


Fig. 2.



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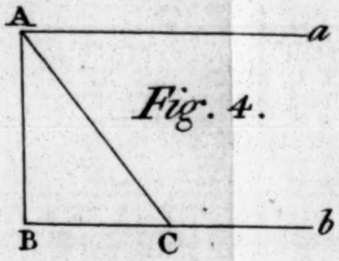

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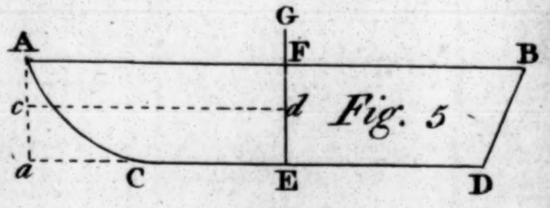


Fig. 5

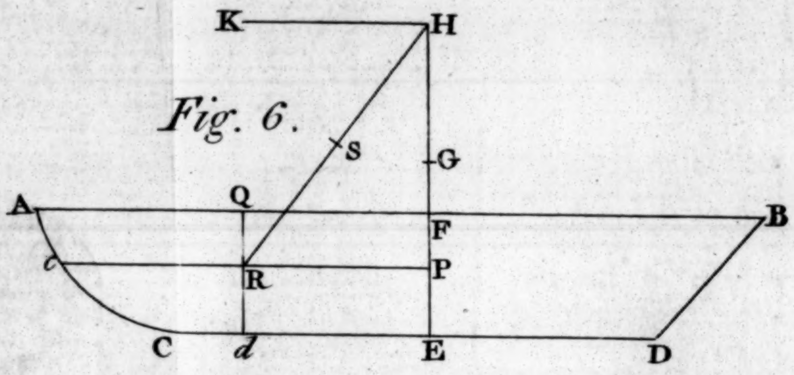
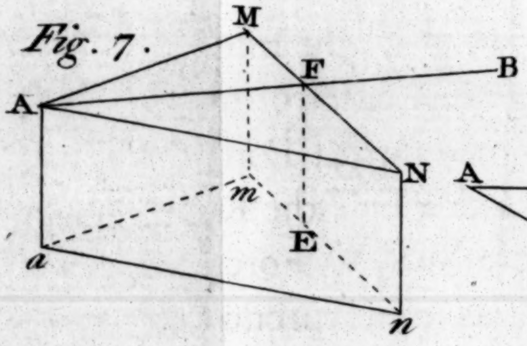


Fig. 6.

Fig. 7.



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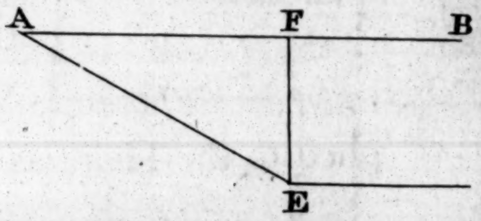


Plate II. Book 2.

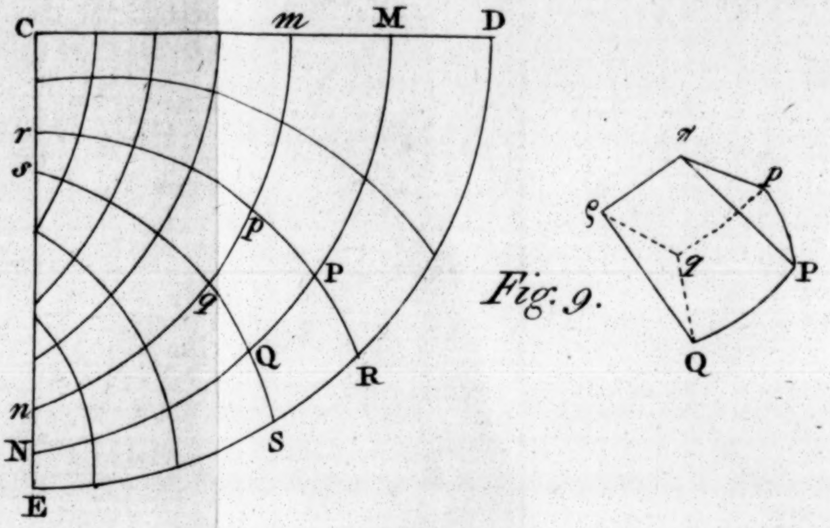
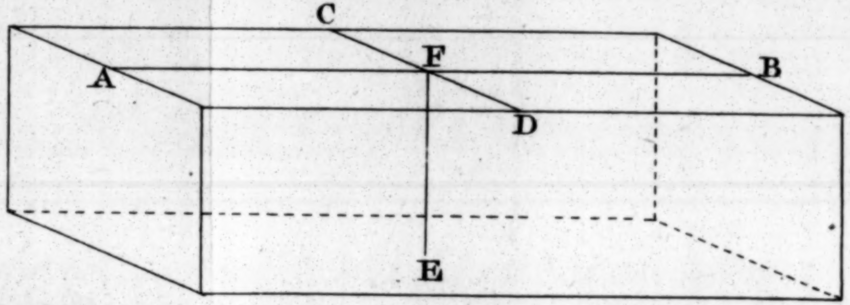


Fig. 9.

Fig. 10.



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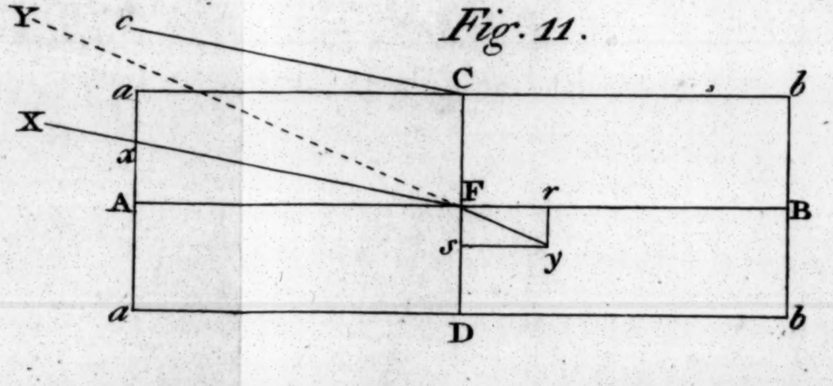
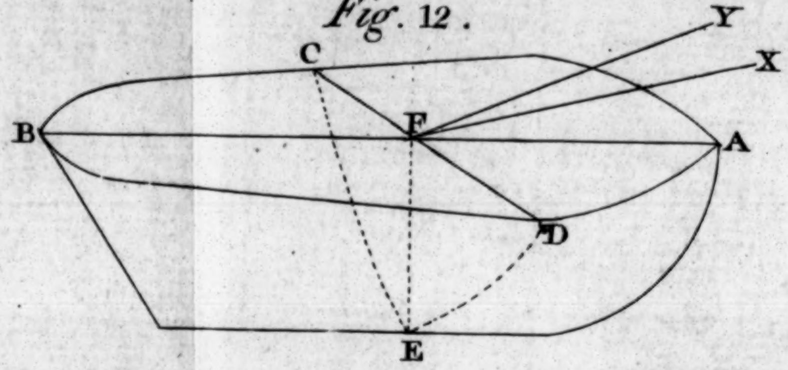


Plate III. Book 2.

Fig. 12.



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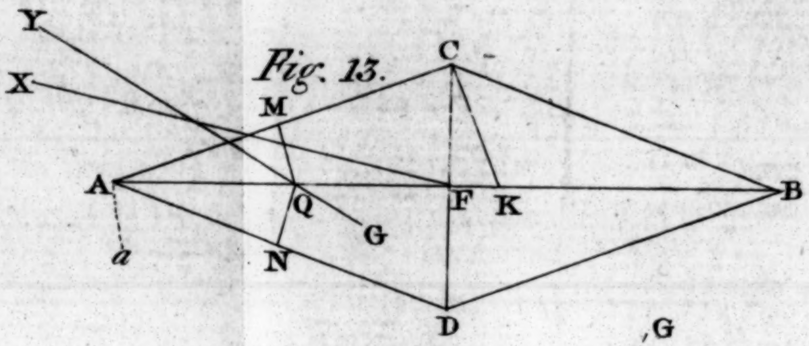


Fig. 14.

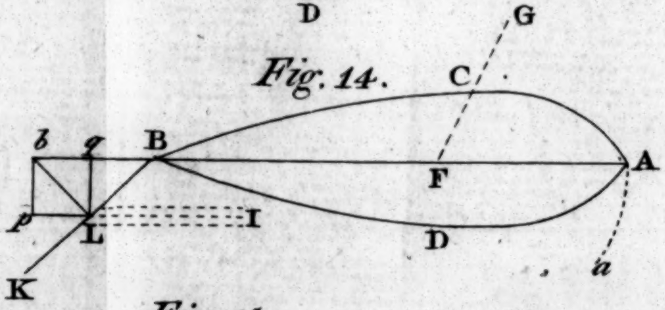


Fig. 15.

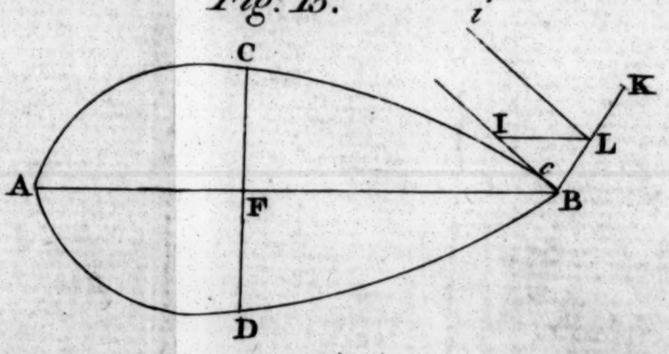
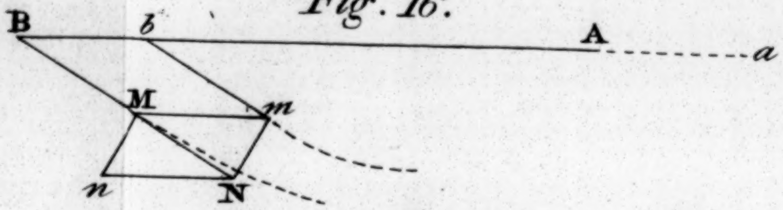


Plate IV. Book 2.

Fig. 16.



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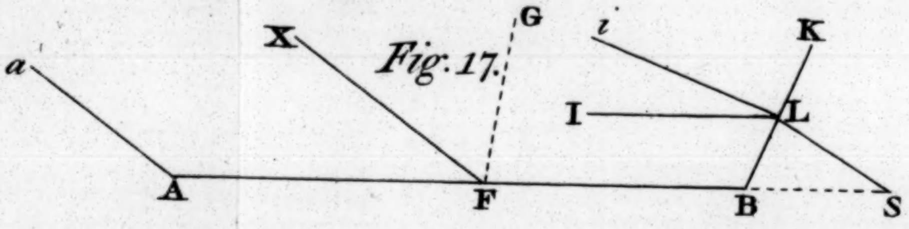


Fig. 18.

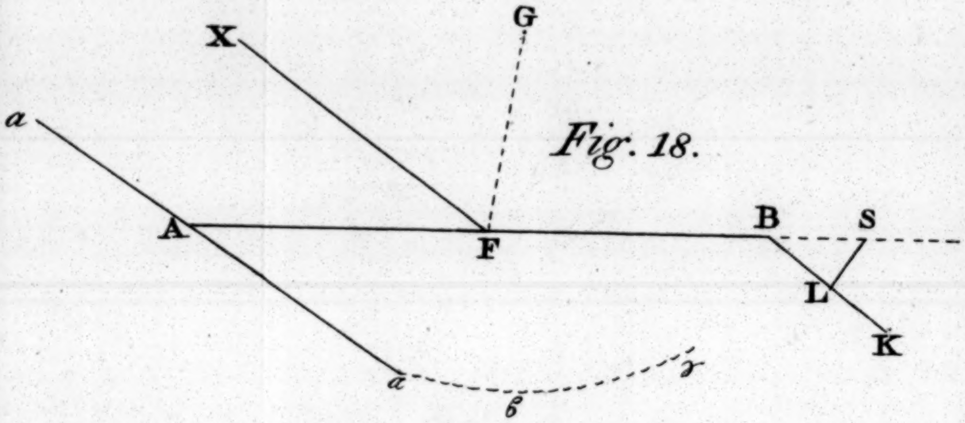
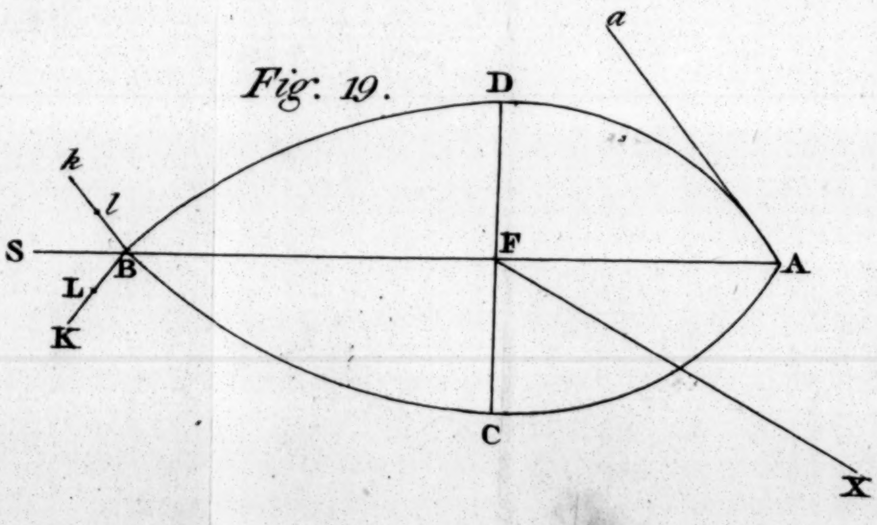


Fig. 19.



that this Momentum is, 1ſt, proportional to the Square of the Velocity of the Veſſel; 2dly, to the Surface of the Rudder; 3dly, to the Interval B L; and, 4thly, to the Square of the Sine of Obliquity, in which it is required to maintain the Rudder. CHAP. IX.

BOOK III.

Upon the Maſts and of the Management of Veſſels.

CHAP. I.

Upon the Sails and the Force of the Wind.

§ 1.

THE Air is a Fluid Matter like Water but much more ſubtile. Experiments have ſhewn us that the Denſity of Water is about 800 times greater than that of Air, therefore the Weight of a Volume of Air will be only one 800th Part of the ſame Volume when filled with Water: conſequently when the Air ſtrikes any Surface whatever with a certain Velocity, its Effect will be only one 800th Part of that which would ariſe from the Water ſtriking it with the ſame Velocity; from hence, we are able to aſſign the Force which a Plane Surface $= f^2$ muſt ſuſtain from the Wind acting upon it in a perpendicular Manner with a given Velocity $= c$: for having already ſeen that if the ſame Surface was ſtruck by the Water, the Force would be equal to the Weight of a

Volume of Water expreſſed by $\frac{c^2 f^2}{4g}$, we conclude that the Force of the Wind will be equal to the Weight of a Volume of Water expreſſed by $\frac{c^2 f^2}{800 . 4g}$, where it muſt be remembered that g denotes the Height which a heavy Body will fall in one Second of Time, and that we always meaſure Velocities by the Spaces which would be run through in one Second. Therefore, ſince we have determined the Force of the Reſiſtance by the Weight of a Volume of Water, we are alſo able to expreſs the Force of the Wind in the ſame Manner.

§ 2.

As it is not ſo eaſy to obſerve the true Velocity of the Wind or the Space which it paſſes over in one Second of Time, and beſides the Wind may change every Moment, the late Mr. *Bouguer* has contrived a very ſimple Inſtrument, by means of which we may immediately diſcover the Force which the Wind exerts upon a given Surface. This is a hollow Tube A A B B in which a Spiral Spring C D is fixed that may be more or leſs compreſſed by a Rod F S D paſſing through a Hole within the Tube at A A; then having obſerved to what Degree different Forces or given Weights

Plate I. Fig. 1.

are capable of compreſſing the Spiral, we put Diviſions upon the Rod in ſuch a Manner that what we obſerve at S indicates the Weight requiſite to force the Spring into the Situation C D: afterwards we join in a perpendicular Manner to this Rod at F, a plane Surface E F E of a given Area either greater or leſs as we may judge proper; then nothing more is neceſſary than to oppoſe this Inſtrument to the Wind in order that it may ſtrike the Surface in the Directions V E, V E parallel to that of the Rod, and the Mark at S, will ſhew the Weight to which the Force of the Wind is equivalent; from thence it will be eaſy afterwards to reduce this Weight to a Volume of Water as we have hitherto expreſſed all Forces. Therefore when the ſame Wind blows in a perpendicular Manner againſt any plane Surface whatever, we may always readily determine the Force which it exerts.

§ 3.

It is the ſame thing when the Wind ſtrikes a plane Surface obliquely, and the Force is diminiſhed in the Ratio of the Square of the Sine of the Obliquity. Thus, if the Surface was $= f^2$, the Velocity of the Wind $= c$, and the Obliquity $= \theta$, the Force would be equal to the

Weight of a Maſs of Water, whoſe Volume is $\frac{1}{800} . \frac{cc}{4g} . f^2 . \text{Sin. } \theta^2$.

But the Direction of this Force is always perpendicular to the Surface, and paſſes through its Center of Gravity. From hence we ſhall eaſily know the Force which the Wind exerts upon the Sails; and as the Sails are ſo extended that their Surface may be regarded as a Plane whoſe Area is $= f^2$, the Expreſſion which we have juſt given will expreſs the Force of the Wind. In order to form a more ſatisfactory Idea, let us conſider ſuch a Sail whoſe Surface we ſhall ſuppoſe equal to a Foot Square or $f^2 = 1$, if this Sail be ſtruck by the Wind in a perpendicular Manner, and with a Velocity equal to ten Feet in one Second, then our Expreſſion will give $\frac{1}{800} . \frac{100}{64}$ Cubic Feet becauſe $g = 16$; which Fraction is $= \frac{1}{512}$ or in Decimal Parts = 0,00195. Hence if the Wind moved with the Velocity of 20 Feet in a Second, the Force would be $= \frac{1}{128}$ of a Cubic Foot; if the Wind moved thirty Feet in a Second, this Force would be $= \frac{1}{57}$ and if it moved 40 Feet in a Second, the Force would be the $\frac{1}{32}$ Part of a Cubic Foot, which is equivalent to ſomething more than the Weight of Two Pounds Averdupoize.

§ 4.

But we can never Set the Sails in ſuch a Manner that their Surfaces will become Planes, eſpecially when the Wind is very ſtrong and ſtrikes almoſt perpendicularly upon them; for in this Caſe the Sails are more or leſs curved according to a Figure which Mathematicians have truly determined. But it is of very little Importance for our Purpoſe to know this Figure, and it is ſufficient to remark, that the more a Sail is curved the more alſo will the Force of the Wind be diminiſhed, and for the very ſame Reaſon that the Fore Part of a Veſſel which is either curved or ſharp will ſuffer leſs Reſiſtance than one that is flat. We have found even that if the Curvature of a Sail ſhould approach to that of a Hemiſphere, the Force of the Wind would be reduced to one Half of that which a great Circle of the ſame Sphere would experience; and as the Surface of a great Circle is but half that of the Hemiſphere, it follows that a Sail curved into a Hemiſphere would only receive one fourth Part of the Force of the Wind which it would receive upon its plane Surface. It is therefore neceſſary to employ every Means for to hinder or at leaſt diminiſh the Curvature of the Sails as much as Circumſtances will admit of. But as we may always conceive a plain Sail which would produce the ſame Force as a curved one, we ſhall not embarraſs ourſelves any longer upon the Curvature of Sails; and in the following Reſearches we ſhall conſider them all as perfect Planes, by ſuppoſing them leſs in Proportion.

§ 5.

Hitherto we have conſidered the Sail as at Reſt, but ſince it has the ſame Motion as the Veſſel above which it is extended, there will often reſult as great a Change in the Force as the Wind exerts upon it. Let us ſuppoſe the Sail to be carried in a certain Direction with the Velocity $= v$, and that the Wind blows in the ſame Direction with a greater Velocity $= c$; it is evident that the Wind will act in the very ſame Manner upon the Sail as if the Sail was at Reſt, and the Wind ſhould ſtrike it with a Velocity $= c - v$; but if the Velocity of the Wind c was leſs than that of the Sail, it would be ſtruck by it on the oppoſite Side; and if the Direction of the Wind was contrary to the Motion of the Sail, the Shock would be made with a Velocity $= c + v$; from whence we ſee that in Navigation it is very neceſſary to diſtinguiſh the true Velocity and the true Direction of the

Wind from that with which it acts upon the Sails. For this Purpoſe, we ſhall call the Wind which acts upon the Sails in Motion, the apparent Wind, in order to diſtinguiſh it from the true Wind, with which the Sails would be ſtruck if they were at Reſt.

§ 6.

Plate I. Fig. 2.

In order to explain this Difference in general, let the Line S T repreſent the Direction and Velocity with which the Sail is carried, or let this Line S T denote the Space which it moves through in one Second: alſo ſuppoſe the Wind to blow in the Direction V S with a Velocity expreſſed by this ſame Line V S, which conſequently repreſents the true Wind. Now it is required to find the apparent Wind or that which would act upon the Sail at Reſt, in the ſame Manner as the true Wind acts upon the Sail in Motion. In order to reſolve this Queſtion, let us conceive the whole Syſtem to have a Motion contrary and equal to that of the Wind, or that the whole is carried in the Direction S V with a Velocity repreſented by this ſame Line; then the Air will in this Caſe be reduced to Reſt, and the Sail will have a Motion compoſed of its proper Motion S T, and of its impreſſed Motion S V. We have therefore only to compleat the Parallelogram S T v V, whoſe Diagonal S v will repreſent the Motion of the Sail in a Calm, and therefore reciprocally the Sail will ſuſtain the ſame Force as if it was at Reſt, and the Wind ſtruck it according to the Direction, and with the Velocity v S: Conſequently this Diagonal v S will ſhew us preciſely that which we have named the apparent Wind.

§ 7.

Therefore if the Line S T expreſſes the Line of Direction in which the Sail moves, and the Line V S that of the true Wind, the apparent Wind will be repreſented by the Diagonal v S; but in order to find its Effect upon the Sail, we have only to conceive the Sail as being at Reſt, and that it is ſtruck by the Wind denoted by this Line v S; then by making uſe of the Expreſſions which have been already determined, it will be eaſy to find the Force which the true Wind V S exerts upon the Sail moving in the Direction S T. For if we call the Velocity of the Sail S T $= v$, the Velocity of the true Wind V S $= c$, and the Angle V S T $= \zeta$; the Velocity of the apparent Wind v S will be

$$\sqrt{c^2 - 2cv \,.\, \text{Coſ.}\,\zeta + v^2}$$

and for the Direction we find

CHAP. I.

$$\text{Sin. } v\,S\,T = \frac{c\ \text{Sin. } \zeta}{\sqrt{c^2 - 2\,c\,v\,.\,\text{Coſ. } \zeta + v^2}};$$

or rather

$$\text{Tang. } v\,S\,T = \frac{c\,.\,\text{Sin. } \zeta}{v - c\,.\,\text{Coſ. } \zeta}.$$

From thence knowing the Velocity of the apparent Wind v S, which ſuppoſe $= u$ with the Velocity of the Sail S T $= v$, and alſo the Angle v S T $= \eta$, we ſhall readily determine that of the true Wind, or

$$c = \sqrt{u^2 - 2\,u\,v\,.\,\text{Coſ. } \eta + v^2}$$

$$\text{and Tang. } \zeta = -\frac{u\,.\,\text{Sin. } \eta}{v - u\,.\,\text{Coſ. } \eta}.$$

§ 8.

It is proper to remark here, that thoſe who are on Board a Veſſel in Motion, never obſerve the true Wind, but always the apparent Wind, even the Vanes and Flags conſtantly ſhew this apparent Wind; and the forementioned Inſtrument will in the ſame Manner only indicate the Velocity of the apparent Wind. Thus when it is required to determine the Force which the Wind exerts upon the Sails of a Veſſel, we have only to obſerve on Board both its Direction and Velocity, and we ſhall then have the apparent Wind ſought, which acts in the ſame Manner upon the Sails as if they were at Reſt, then by Means of the Formulas before given the Force itſelf will be known. This difference between the true and apparent Wind will explain a Phenomenon which often appears to us very Paradoxical: It is, that two Veſſels which paſs each other at Sea obſerve different Winds, altho' the true Wind blows equally upon both. For let S T repreſent the Direction and Velocity of one of theſe Veſſels, and S′ T′ the Direction and Velocity of the other, at the ſame time that both are ſtruck by the ſame true Wind V S or V′ S′, then having drawn the Diagonals *v* S and *v*′ S′, the Vanes of the firſt Veſſel will ſhew the Wind in the Direction *v* S, and thoſe of the other in that of *v*′ S′, and theſe Directions may often differ from each other by ſeveral Points. Plate I. Fig. 3.

CHAP. II.

Upon the Maſts, and upon the Form of the Fore Part of a Veſſel ſuitable to the Action of the Sails.

§ 9.

IT is not here neceſſary to enter into a Detail of all the Particulars which relate to the Maſts of a Veſſel, or the Manner in which they carry the Sails; it will be ſufficient for our Purpoſe to remark that the whole Space above the Veſſel is filled with Sails as much as poſſible, in order to obtain from every Wind all the Efforts capable of giving Motion to the Veſſel. With this View Maſts are eſtabliſhed to receive Sails through their whole Height, and as much Breadth is given to them as the Size of the Veſſel will admit of. Often alſo Sails are uſed between the Maſts, and both towards the Head and Stern of the Veſſel, in order to increaſe as much as poſſible the Surfaces upon which the Wind may exert its Efforts. But whatever the Number of Maſts may be, and however great the Number of Sails, we may always conceive one Sail only, ſuch, that being ſtruck by the Wind, it would produce the ſame Effect as all the Sails taken together; by which means the Queſtion is reduced to determine both the Size of this Equivalent Sail, and the Place of its Application.

§ 10.

But at firſt we muſt obſerve that the Surface of this Equivalent Sail muſt be equal to the Sum of all the Actual Sails together; and we ſhall conſider the whole as Planes parallel to each other, as there is no reaſon why we ought to ſet the Sails differently with reſpect to the Wind,

Wind; beſides Navigation conſtantly requires that every Sail be equally expoſed to the Wind, except ſome ſmall ones which in particular Caſes may be required to aid the Action of the Rudder. Thus our Equivalent Sail will be always parallel to the Actual Sails, and its Surface equal to the Sum of all their Surfaces together: however, it is ſtill neceſſary to obſerve, that we muſt only add together ſuch Sails as are actually ſtruck by the Wind, and that we muſt exclude all thoſe upon which the Wind cannot freely blow, in conſequence of its being already intercepted by thoſe Sails which are more Aft; and the Diſpoſition of the Sails with reſpect to the Direction of the Wind, will readily ſhew us ſuch as may be uſefully employed: For Example, if the Veſſel runs before the Wind the Sails upon the Mizzen Maſt would receive the full Action of the Wind, whilſt thoſe upon the Main and Fore Maſts would only be partially effected by it.

§ 11.

Having eſtabliſhed the Idea of an Equivalent Sail and its Extent, and ſince all the Force which it would receive from the Action of the Wind muſt always paſs through the Center of Gravity of its Surface, and be perpendicular thereto; this Point muſt without doubt be of the laſt Importance, and it is that which the late Mr. *Bouguer* has named the *Centre Velique*, and is therefore the ſame as the Center of Gravity of the Equivalent Sail, conſequently the Mean Direction of all the Forces with which the Wind acts, muſt paſs through this Point. It is therefore very eſſential to know exactly the Place of this Centre Velique: And at firſt it is evident, that this Center muſt be ſituated ſomewhere in the Continuation of the Diametrical Plane of the Veſſel, ſince commonly all the Sails are equally divided by this Section; hence the Determination of this Point muſt depend upon its Elevation above the Veſſel, or rather above the Level of the Sea, and upon the Place where a Perpendicular let fall from it, would interſect the Principal Axe of the Veſſel. But we have already demonſtrated in the preceding Book, that this Point muſt be ſituated nearer to the Head than the Stern of the Veſſel.

§ 12.

The Situation and Dimenſion of each of the Actual Sails being known, we may from the Principles of Staticks determine the true Place of the Centre Velique. For, let the Surface of any Sail whatever be ſuppoſed $= K$, and the Elevation of its Center of Gravity above the Sea $= b$ which is the Perpendicular drawn from this Point to the Surface of the Sea, alſo ſuppoſe the Diſtance of this Line from the Stern to be $= l$; then if each of the other Sails be reſpectively denoted, by K', h', l'; K'', h'', l''; K''', h''', l'''; &c. reſpectively, the Height of the Centre Velique will be equal to

$$\frac{K\,h + K'\,h' + K''\,h'' + K'''\,h''' + K''''\,h'''' + \&c.}{K + K' + K'' + K''' + K'''' + \&c.}$$

and the Diſtance of this Point from the Stern of the Veſſel will alſo be equal to

$$\frac{K\,l + K'\,l' + K''\,l'' + K'''\,l''' + K''''\,l'''' + \&c.}{K + K' + K'' + K''' + K'''' + \&c.}$$

We ſee evidently that the Height of the Centre Velique muſt principally depend upon the Height of the Maſts, which we cannot increaſe beyond certain Limits, and that theſe Limits muſt depend both upon the Size of the Veſſel and its Deſtination: But ſince the more elevated Sails are commonly much leſs than the lower ones, it is evident that the Centre Velique will not fall in the Mean Height which the Sails occupy, but always ſomething lower. Hence it is eaſy to comprehend how the Actual Sails ought to be diſpoſed, in order that the Centre Velique may fall in a given Point; and therefore we ſhall always conſider both the Place of the Centre Velique, and the Size of the Equivalent Sail as given, in order to determine all the Rules which may conduce

CHAP. II. to render the Management of Veſſels more perfect.

§ 13.

Plate I. Fig. 4. Let us now ſuppoſe the Diametrical Section of any Veſſel to be repreſented by Fig. 4, where the Line A B expreſſes the Water Line or great Axe of the Body, the Line L E H the Keel, the Point G the Center of Gravity of the Veſſel, and the Point W the Centre Velique, more elevated than G by the Interval W *g*, and more advanced towards the Head by the Space G *g* or *F f*. Then ſince the Force of the Wind muſt always paſs through the Centre Velique W, and becauſe the Surface of the Equivalent Sail is commonly as great as Circumſtances will permit, there muſt in conſequence reſult a very great Momentum of Force tending to produce an Inclination in the Veſſel, which will be ſo much the greater as the Centre Velique is more elevated above the Center of Gravity G. In direct Courſes where we are able to employ the whole Force of the Wind, this Momentum will give the Veſſel an Inclination towards the Head; and altho' the Stability with reſpect to this Inclination is the greateſt, yet the Motion of the Veſſel will always be conſiderably affected by it. In order to remedy this Inconvenience, it is therefore neceſſary that the Reſiſtance of the Fore Part of the Veſſel ſhould furniſh a like

Momentum of Force in an oppoſite Direction, which will happen when the mean Direction of the Reſiſtance paſſes through the Centre Velique W. For ſuppoſe W R to be the Force of the Reſiſtance, and that we reſolve it into two others, of which W *s* is Horizontal, and W *u* Vertical; the former muſt evidently be entirely deſtroyed by the Wind, and conſequently there will no longer reſult any Momentum for to incline the Veſſel: But the Vertical Force W *u* will even produce a double good Effect, by lifting the Veſſel and diminiſhing its Weight; therefore the Depth of the Body muſt likewiſe be diminiſhed by it, and as this Force is applied before the Center of Gravity G, it muſt alſo tend to elevate the Fore Part of the Veſſel: by which Means if the Centre Velique was even more elevated, there would not be any thing to apprehend from it. Hence we ſee that this good Effect might be ſtill increaſed by giving to the Sails themſelves ſome ſmall Inclination to the Horizon, in order that the Force of the Wind may act ſomething upwards.

§ 14.

But the Mean Direction of the Reſiſtance would always infallibly paſs through the Centre Velique W, if the Surface of the Fore Part of the Veſſel was a Portion of a Sphere deſcribed from the Center W with the Radius W H or

W A: For ſince all the perpendicular Directions upon the Surface of the Fore Part would then paſs through the ſame Point W, all the Elementary Efforts of the Water would therefore be re-united in the ſame Point, and conſequently alſo their Mean Direction would be W R; and this muſt equally happen both in Direct as well as Oblique Courſes, becauſe the [illegible] Portion of the Sphere would ſtill receive the Shock of the Water. However altho' other Circumſtances will not allow ſuch a Conſtruction for the Fore Part of a Veſſel, it will ſtill be always very advantageous that the Rake of the Stem itſelf H A be an Arc of a Circle deſcribed from the Centre Velique W. From whence we ſhall be furniſhed with a very eaſy Rule for to give to the Stem H A the moſt proper Form from the Keel H to the Extremity of the Body A: With reſpect to the Portion above the Water, it will be always neceſſary to give it a Direction nearly Vertical, in order that the Waves of the Sea may in Tempeſtuous Weather have the leſs Power to agitate the Veſſel.

§ 15.

With reſpect to Direct Courſes, we might obtain the ſame Advantages, provided the Fore Part of the Veſſel had the Figure of a round Solid, generated by the Rotation of ſome Figure about the Point W, or rather, round a horizontal Axe paſſing through this Point and parallel to the tranſverſe Axe of the Veſſel. Therefore in order to give to the Fore Part of the Veſſel ſuch a Form, ſuppoſe the Veſſel to be cut by a Plane, paſſing through the Point W and the Extremity H of the Keel, and parallel to the tranſverſe Axe of the Veſſel, then this Section being turned round the ſaid Axe conceived at W, will deſcribe the true Figure of the entire Fore Part of the Veſſel, or at leaſt of all that Part which is below the Surface of the Water; for in reſpect to the Part above the Water, we may give it ſuch a Form as may beſt anſwer our Purpoſe.

§ 16.

Altho' ſuch a Figure is properly adapted to direct Courſes only, we may ſtill derive from it very great Advantages in Oblique Courſes, notwithſtanding the Momentum to incline the Veſſel be not entirely deſtroyed; as there will remain but a very ſmall Portion to give it an Inclination on one Side; and even this Momentum which reſults from the Force of the Wind inclining the Veſſel to Leeward, will be ſo much the leſs conſiderable, the more the Wind ſtrikes it obliquely, which will conſtantly happen when the Veſſel is upon a Wind. However, it is always of the laſt Importance to augment as much as poſſible the Stability of Veſſels

with

CHAP. II. with reſpect to their great Axe, and we have already ſhewn how this End is to be attained in the moſt eaſy Manner, by increaſing the Breadth of the Body with reſpect to the Draught of Water; the Lengthening of Veſſels may alſo contribute much to it, ſeeing that we may gain ſome Diminution in the Depth of the Body, becauſe the whole Weight of the Veſſel is ſuppoſed to remain the ſame, or to receive a leſs Increaſe than the Length.

§ 17.

The Conſtruction which we have juſt given, for the Rake of the Stem is ſo ſimple and ſo eaſy to execute, that we can deſire nothing further. However, it will be proper to add ſome Rules by which we may determine the Projection and Obliquity of that part of the Stem which is below the Water. For this Purpoſe let us ſuppoſe as before, the Length of the Body Plate I. Fig. 4. $AB = a$, the Breadth $= b$, and the Depth or Draught of Water $EF = e$: Alſo ſuppoſe the Elevation of the Centre Velique W above the Surface of the Water, or $Wf = h$; then we have ſeen that it is neceſſary to take the Interval $Af = \frac{2}{5}a$, therefore $Ff = \frac{1}{10}a$ by ſuppoſing the Point F to be in the Middle of Axe AB, from thence we ſhall have $We = h + e$

and

and $E\,c = \frac{1}{10}\,a$; now the Right-angled Triangle $A\,W\,f$ gives $AW^2 = b^2 + \frac{4}{25}\,a^2$, which is equal to the Square of the Line W H, and by taking from this the Square of the Height $W\,e = b^2 + 2\,b\,e + e^2$, we ſhall have $H\,e^2 = \frac{4}{25}\,a^2 - 2\,b\,e - e^2$ and therefore

$$E\,H = \frac{1}{10}\,a + \sqrt{\frac{4}{25}\,a^2 - 2\,b\,e - e^2}.$$

from whence we know the Extremity H of the Keel, or the Beginning of the Stem H A; and by taking this Interval from the demi Axe $A\,F = \frac{1}{2}\,a$, we ſhall obtain the Projection of the Stem as far as the Water Line, or

$$A\,b = \frac{2}{5}\,a - \sqrt{\frac{4}{25}\,a^2 - 2\,b\,e - e^2}.$$

§ 18.

Let us now apply this Expreſſion to the different Species of Veſſels which are at preſent in Uſe: And at firſt it ſeems that we may put in general $W\,f$ or $b = 4\,e$, and the whole Heighth $W\,e = 5\,e$, ſince a ſmall Difference in this Element is not of any Conſequence; afterwards we ſhall ſuppoſe the Breadth $b = \frac{5}{2}\,e$ as the

Stability requires, and $a = n b = \frac{5}{2} n e$, where the Number n repreſents the different Species of Veſſel. This being determined the Projection of the Stem will then become

$$A b = n e - e \sqrt{n^2 - 9}.$$

By writing therefore for n the Numbers 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, and 6 ſucceſſively, we ſhall have for the ſeven Species of Veſſel the following Projections of the Stem.

I. If $a = 3\,b$, we ſhall have
$Ab = 3\,e$, and
$a = 7\frac{1}{2}\,e$.

II. If $a = 3\frac{1}{2}\,b$, we ſhall have
$Ab = 3{,}5\,e - e\sqrt{3{,}25} = 1{,}691\,e$, and
$a = 8\frac{3}{4}\,e$.

III. If $a = 4\,b$, we ſhall have
$Ab = 4\,b - \sqrt{7} = 1{,}355\,e$, and
$a = 10\,e$.

IV. If $a = 4\frac{1}{2}\,b$, we ſhall have
$Ab = 4{,}5\,e - e\sqrt{11{,}25} = 1{,}146\,e$, and
$a = 11\frac{1}{4}\,e$.

V. If $a = 5\,b$, we ſhall have
$Ab = 4{,}5\,e - e\sqrt{16} = e$, and
$a = 12\frac{1}{2}\,e$.

VI. If $a = 5\frac{1}{2}\,b$, we ſhall have
$Ab = 5{,}5\,e - e\sqrt{21{,}25} = 0{,}891\,e$, and
$a = 13\frac{3}{4}\,e$.

VII. If $a = 6\,b$, we ſhall have
$Ab = 6\,e - e\sqrt{27} = 0{,}812\,e$, and
$a = 15e$.

CHAP. II.

Hence we ſee that the more a Veſſel is increaſed in Length, the leſs ought to be the Projection of the Stem, unleſs we are deſirous of increaſing the Height of the Centre Velique W at the ſame Time.

CHAP. III.

Upon the Motion of Veſſels in their Direct Courſes.

§ 19.

IN order that a Veſſel may continue in its proper Courſe, it is neceſſary that the moving Force ſhould act according to the ſame Direction, and therefore all the Sails ought to be diſpoſed in ſuch a Manner as to have their Planes perpendicular to the principal Axe of the Veſſel, by this Means the Directions of the Forces which they receive from the Wind will be all parallel to the ſame Axe. Suppoſe therefore A B to be the great Axe of the Veſſel, A the Head, and B the Stern, and that the Line S F *s* perpendicular to this Axe repreſents the Equivalent Sail, whoſe Surface ſuppoſe $= f^2$. Now the Veſſel in this Situation will be put in Motion according to its proper Direction B A, whenever the

Plate II. Fig. 5.

the Wind acts upon the Aft Part of the Sail S F *s*, which will always happen when the Direction of the Wind V F makes an Acute Angle on either Side. For it is evident that as ſoon as the Angle B F V becomes a right one, the Sail will no longer be affected by the Wind, and if the Wind ſhould come before the Beam and ſtrike the Fore Part of the Sail in a Direction U F, the Veſſel would then be urged Stern Foremoſt.

§ 20.

Let us at firſt ſuppoſe the Angle B F V to vaniſh, or that the Veſſel goes before the Wind, the true Velocity of which is $= c$; and let us alſo ſuppoſe that the Veſſel has already acquired a Velocity $= v$ in the Direction B A itſelf: Therefore ſince the Sail S F *s* has the ſame Motion, it can only be ſtruck by the Wind with the Exceſs of its Velocity above that of the Veſſel, or rather the apparent Velocity will be $= c - v$, and the Direction being perpendicular to the Sail, the Force which will reſult therefrom will be equal to the Weight of a Maſs of Water, whoſe Volume is equal to

$$\frac{1}{800} \cdot \frac{\overline{c - v}^2}{4g} \cdot f^2,$$

and with which the Motion of the Veſſel would be accelerated if it did not meet with any Reſiſtance.

§ 21.

We ſhall now ſuppoſe that the Veſſel meets the ſame Reſiſtance from the Water which a Plane Surface $= r^2$ would experience by moving in a direct Manner therein, with the ſame Velocity v, from whence there would reſult a Reſiſtance $= \frac{v^2}{4g} . r^2$. It is then evident that if the acting Force was greater than this Reſiſtance the Motion would be accelerated, and if it was leſs, the Motion would be retarded. Therefore in order that the Veſſel ſhould run before the Wind with an uniform Velocity, it is neceſſary that the acting Force be equal to the Reſiſtance; hence ariſes this Equation

$$\frac{1}{800} . \frac{\overline{c - v}^2}{4g} . f^2 = \frac{v^2}{4g} . r^2 .$$

$$\text{or, } \overline{c - v} . f = v r . \sqrt{800} .$$

from whence we find the Velocity of the Veſſel

$$v = \frac{c f}{f + r . \sqrt{800}} .$$

ſo that in the Caſe we have juſt conſidered, the Veſſel will receive a Velocity which will be always much leſs than the Velocity of the Wind c, but let us now ſee what the Surface of the Sails ought to be, in order that the Veſſel may acquire either one half, one third, or any other Part of the Velocity of the Wind.

Suppoſe 1ſt, $v = \frac{1}{2} c$, we ſhall then have $f = r . \sqrt{800}$ and $f^2 = 800 . r^2$; but if $v = \frac{1}{3} c$, we then have $f = \frac{1}{2} r . \sqrt{800}$ and $f^2 = 200 . r^2$; and if $v = \frac{1}{4} c$, we ſhall then have $f = \frac{1}{3} r . \sqrt{800}$ and $f^2 = \frac{800}{9} . r^2 = 88 \frac{8}{9} . r^2$; alſo if we required a Velocity $v = \frac{2}{3} c$, we ſhould then find $f = 2 r . \sqrt{800}$ and $f^2 = 3200 . r^2$; hence it is evident, that as ſoon as we have gained a certain degree of Velocity, it will then be uſeleſs to increaſe the Size of the Sails in order to obtain a Velocity which is more conſiderable.

§ 22.

But let us now conſider any Wind whatever which blows in the Direction V F, with the true Velocity $= c$, under the Obliquity $BFV = \theta$, and ſuppoſe the Velocity of the Veſſel to be $= v$. In order to find the apparent Velocity of the Wind, let us alſo ſuppoſe the Right Line V F to repreſent the Velocity c, and that we take $FT = v$, then by compleating the Parallelogram $VFTv$, the Diagonal vF will repreſent the

Plate II. Fig. 6.

CHAP. III. the apparent Wind which acts upon the Sail. But we have ſeen before that when the Angle B F V = θ, we ſhall have

$$vF = \sqrt{c^2 - 2cv \,.\, \text{Coſ.}\, \theta + v^2};$$

and ſince the Angle F T $v = \theta$ and T v = V F = c by letting fall the Perpendicular $v\,u$ upon T F we ſhall have

$$v\,u = c \text{ Sin. } \theta, \text{ and } T\,u = c \,.\, \text{Coſ. } \theta$$

and therefore F $u = c$. Coſ. $\theta - v$, hence we find

$$\text{Tang. } B\,F\,v = \frac{v\,u}{F\,u} = \frac{c \,.\, \text{Sin. } \theta}{c \,.\, \text{Coſ. } \theta - v},$$

and conſequently

$$\text{Sin. } B\,F\,v = \frac{c \,.\, \text{Sin. } \theta}{\sqrt{c^2 - 2cv \,.\, \text{Coſ. } \theta + v^2}}, \text{ and}$$

$$\text{Coſ. } B\,F\,v = \frac{c \text{ Coſ. } \theta - v}{\sqrt{c^2 - 2cv \,.\, \text{Coſ. } \theta + v^2}}.$$

§ 23.

Plate II. Fig. 5. Suppoſe v V to repreſent the apparent Wind with which the Sail is actually ſtruck, then ſince its Velocity has been found

$$vF = \sqrt{c^2 - 2cv \,.\, \text{Coſ.}\, \theta + v^2},$$

and the Coſine of the Angle

B F v

$$\mathrm{B\,F}\,v = \frac{c\,.\,\mathrm{Cof.}\,\theta - v}{\sqrt{c^2 - 2\,c\,v\,.\,\mathrm{Cof.}\,\theta + v^2}},$$

this Coſine will be the Sine of Incidence, or the Sine of the Angle v F S, which being multiplied by the Velocity will give a Product $= c\,.\,\mathrm{Cof.}\,\theta - v$ whoſe Square divided by $4\,g$ muſt be ſtill multiplied by the Surface of the Sails $= f^2$ and divided by 800 in order to obtain the acting Force which will therefore be equal to

$$\frac{1}{800}\,.\,\frac{\overline{c\,.\mathrm{Cof.}\,\theta - v}^2}{4\,g}\,.\,f^2.$$

which being equal to the Reſiſtance $\frac{v^2}{4\,g}\,.\,r^2$ as before, we then find

$$v = \frac{c\,f\,.\,\mathrm{Cof.}\,\theta}{f + r\sqrt{800}}.$$

From hence we ſee that this Expreſſion does not differ from the preceding one, except that in place of c we have here $c\,.\,\mathrm{Cof.}\,\theta$; which a little Conſideration would have made manifeſt at firſt: For the true Wind V F being reſolved into the Directions F B and F S; it is evident that the firſt which is $c\,.\,\mathrm{Cof.}\,\theta$ muſt be diminiſhed by the Velocity v of the Veſſel; therefore the Velocity v of the Veſſel is always not only much leſs than the Velocity c of the Wind itſelf, but ſtill leſs than $c\,.\,\mathrm{Cof.}\,\theta$.

§ 24.

From what has been delivered, it appears that the more the Obliquity of the Wind or the Angle B F V approaches to a right Angle; the leſs will the Velocity of the Veſſel become, and that in the Ratio of the Coſine of this Angle. But it is very neceſſary to remark that we here ſuppoſe the Quantity f^2, or the Surface of all the Sails ſtruck by the Wind to remain the ſame. However upon this Occaſion we meet with a great Paradox, viz. that an Oblique Wind will communicate a greater Velocity to the Veſſel than a Direct Wind in the Direction B F: But this happens when the Veſſel contains ſeveral Maſts furniſhed with Sails; for then it is evident when the Wind blows in the Direction B F, it can only ſtrike upon the Sails of the Maſt next the Stern, by this Means thoſe belonging to the Maſts which are more Forward will become entirely uſeleſs: But when the Wind has any Obliquity it will then ſtrike thoſe Sails which are Forward either entirely or in part; from whence it is eaſy to conceive that the Diminution cauſed by the Obliquity, may be amply recompenſed by the greater Number of Sails put into Action: Therefore it becomes neceſſary in this Caſe always to give to the Quantity f^2 its juſt Value, by eſtimating the whole Surface which is actually ſtruck by the Wind, and thus the more the Obliquity increaſes the more alſo will the Value of f^2 be augmented according to Circumſtances, which the Number of Maſts and their Diſtances from each other will eaſily ſhew us.

§ 25.

After this general Expoſition let us make the Application to the different Species of Veſſels now in Uſe, and let us ſuppoſe as heretofore, the Length of the Body A B $= a$, the Breadth $= b$ and the Depth $= e$; then we have ſeen that the Value of r^2 which repreſents the Area of a Plane Surface that would experience the ſame Reſiſtance in the Water as our Veſſel actually meets with, is nearly (ſee § 33 Book 2d)

$$r^2 = \frac{3}{4} b\, e \cdot \frac{2\, b^2}{a^2 + 2\, b^2}.$$

And with reſpect to the Sails we ſhall as heretofore ſuppoſe the Height of the Centre Velique above the Surface of the Water to be $= 4\, e$; from which it is neceſſary to deduct the Diſtance e nearly in order to have its Elevation above the upper Deck, this Elevation will therefore be $= 3\, e$, and the Height of the Sails will be nearly $= 6\, e$, but ſince the Breadth of the Sails is regulated by the Breadth of the Veſſel, we may ſuppoſe the Surface of the Sails of one Maſt to be $= 6\, b\, e$; alſo becauſe in general we have $e = \frac{2}{5} b$, this Surface will be $= \frac{12}{5} b^2$. But as it is neceſſary to make an Allowance for the Curvature of the Sails by which the Action of the Wind is leſſened, we ſhall only ſuppoſe the Surface of the Sails for one Maſt to be $= 2 b^2$ and therefore for the three principal Maſts we ſhall have a Surface $= 6 b^2$. Hence it is evident, that when the Ship runs before the Wind, we cannot eſtimate the Value of f^2 at more than $2 b^2$, but when the Wind acts obliquely, this Value may be increaſed to $6 b^2$. We ſhall therefore ſuppoſe in general $f^2 = \alpha . b^2$.

§ 26.

Having therefore $f^2 = \alpha . b^2$, and

$$r^2 = \frac{3}{4} . b e . \frac{2 b^2}{a^2 + 2 b^2},$$

or rather becauſe $e = \frac{2}{5} b$

$$r^2 = \frac{3}{10} . b^2 . \frac{2 b^2}{a^2 + 2 b^2} = \frac{3 b^4}{5 a^2 + 10 b^2}.$$

If we ſubſtitute theſe Values in the Expreſſion found (§ 23) for the Velocity v of the Veſſel; when the Wind blows in the Direction V F with a Velocity $= c$, its Obliquity B F V being $= \theta$, we ſhall then find

$v =$

$$v = \frac{c \cdot \text{Coſ.}\, \theta \cdot \sqrt{\alpha}}{\sqrt{\alpha} + \frac{b \cdot \sqrt{480}}{\sqrt{a^2 + 2b^2}}}$$

but it is neceſſary to remark that the Number α may increaſe from 2 to 6, and as a^2 is always much greater than $2b^2$, then becauſe this Reſearch is not ſuſceptible of Preciſion, we may boldly ſuppoſe

$$v = \frac{ac \cdot \text{Coſ.}\, \theta \cdot \sqrt{\alpha}}{a\sqrt{\alpha} + b\sqrt{480}}$$

From whence we conclude that when the Wind is right Aft or $\theta = 0$ and $\alpha = 2$, we ſhall have

$$v = \frac{ac}{a + b\sqrt{240}} = \frac{2ac}{2a + 31 \cdot b}.$$

This Expreſſion will furniſh us with the following Velocities for the ſeven principal Species of Veſſel, which the Wind acting with a Velocity c will communicate to them.

I. If $a = 3\,b$, we ſhall have $v = \frac{6}{37}\,c$.

II. If $a = 3\frac{1}{2}\,b$, $v = \frac{7}{38}\,c$.

III. If $a = 4\,b$, $v = \frac{8}{39}\,c$.

IV. If $a = 4\frac{1}{2}\,b$, $v = \frac{9}{40}\,c$.

V. If $a = 5\,b$, $v = \frac{10}{41}\,c$.

VI. If $a = 5\frac{1}{2}\,b$, $v = \frac{11}{42}\,c$.

VII. If $a = 6\,b$, $v = \frac{12}{43}\,c$.

§ 27.

But when the Wind has any Obliquity $BFV = \theta$, in order to find the Velocity of the Veſſel, it is neceſſary not only to multiply the Velocity of the Wind c by the Coſine of θ, but alſo to give to the Letter α a greater Value than 2; becauſe a greater Surface of Sail will then be acted upon by the Wind: from whence it is obvious that an Oblique Wind may communicate to the Veſſel a greater degree of Velocity than one which is Direct, and it will be eaſy to aſſign the Velocity of the Veſſel in all Caſes: Alſo when the Wind has a ſmall Obliquity, we may always obtain a greater Advantage by the choice of ſome Oblique Courſe; and by applying the Formulas which have been already delivered, we ſhall be able to reſolve the following Queſtion, which is without doubt of the greateſt Importance in the Art of Navigation.

CHAP. III.

The Direction of the Wind and that of the Courſe which a Veſſel muſt follow being given, to find the Diſpoſition of the Sails, in order that the Veſſel may receive the greateſt Velocity poſſible.

But before the Solution of this Problem is given, it is neceſſary to detail in a more particular Manner every thing that relates to Oblique Courſes.

CHAP. IV.

Upon the Motion of Veſſels in their Oblique Courſes.

§ 28.

A Veſſel will always follow a Courſe more or leſs Oblique, when the acting Force is not communicated in the Direction of the great Axe; and that conſtantly happens when the Sails are not Set perpendicular to that Axe. The Principal Axe of a Veſſel being therefore repreſented by the Line A B, let the Line S F *s* be the Direction of the Equivalent Sail, whoſe Surface we always

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Plate II. Fig. 7.

CHAP. IV.

always ſuppoſe $= f^2$ and denoting the Obliquity or the Angle A F S by the Letter η, which this Direction makes with A B to Windward, alſo ſuppoſing the Wind to ſtrike upon the Aft Part of the Sail S s, then becauſe the acting Force will be always impreſſed in the Direction of the Line F Y perpendicular to the Sail, therefore ſince the Angle S F Y is a Right one, we ſhall have the Angle A F Y $= 90° - \eta$, which is the ſame as we have heretofore expreſſed by the Letter ψ, where we have aſſigned the Direction of Motion F X itſelf, the Lee-way of the Veſſel A F X being repreſented by the Letter φ; thus it is neceſſary to repeat both the Relation between the two Angles

$$AFX = \varphi \text{ and } AFY = \psi = 90° - \eta$$

as well as the Expreſſion which denotes the Reſiſtance that the Veſſel meets with in ſuch an Oblique Courſe.

§ 29.

The Length of the Body A B being always ſuppoſed $= a$, its Breadth $= b$ and the Draught of Water $= e$, we have found (ſee § 35 Book II.) the Relation between φ and ψ expreſſed by this Equality

$$\text{Tang.}\,\psi = \frac{a^3}{2\,b^3}\,.\,\text{Tang.}\,\varphi^2$$

and if the Velocity of the Veſſel in the Direction

tion F X be ſuppoſed $= v$, we ſhall have the Force of the Reſiſtance (ſee § 38 Book II.) expreſſed by

$$\frac{v^2}{4g} \cdot \frac{3}{4} a e \cdot \frac{\text{Sin. } \varphi^2}{\text{Sin. } \psi} .$$

which muſt be equal to the acting Force, ſince the Motion of the Veſſel is uniform. In order to render theſe Formulas more applicable to Practice, we ſhall add two Tables which indicate for every five Degrees of the Obliquity of the Sails or the Angles A F S $= n$, the Values of the Lee-way or the Angles A F X $= \varphi$, as well as the Value of the Formula $\frac{\text{Sin. } \varphi^2}{\text{Sin. } \psi}$ which ſerves to ſhew the Force of the Reſiſtance.

TABLE I.

Which ſhews the Lee-way AFX = φ *for each of the ſeven Principal Species of Veſſel, when the Obliquity of the Sails or the Angle* AFS = η *is given.*

Angle	Species		
AFS = η	$a = 3b$	$a = 3\frac{1}{2}b$	$a = 4b$
90°	0	0	0
85	4° : 36′	3° : 40′	2° : 59′
80	6 : 31	5 : 11	4 : 14
75	7 : 58	6 : 40	5 : 27
70	9 : 19	7 : 25	6 : 5
65	10 : 30	8 : 24	6 : 54
60	11 : 41	9 : 19	7 : 39
55	12 : 50	10 : 15	8 : 25
50	14 : 1	11 : 11	9 : 12
45	15 : 12	12 : 12	10 : 2
40	16 : 33	13 : 16	10 : 55
35	17 : 26	14 : 30	11 : 57
30	19 : 43	15 : 52	13 : 6
25	21 : 43	17 : 31	14 : 29
20	24 : 17	19 : 42	16 : 20
15	27 : 44	22 : 39	18 : 51
10	32 : 57	27 : 13	22 : 50
5	42 : 37	36 : 9	30 : 52

TABLE I.

Which ſhews the Lee-way AFX $= \varphi$ *for each of the ſeven Principal Species of Veſſel, when the Obliquity of the Sails or the Angle* AFS$=\eta$ *is given.*

Angle	Species			
AFS$=\eta$	$a = 4\frac{1}{2}\,b$	$a = 5\,b$	$a = 5\frac{1}{2}\,b$	$a = 6\,b$
90°	0	0	0	0
85	2°: 30′	2°: 9′	1°: 52′	1°: 37′
80	3 : 33	3 : 2	2 : 38	2 : 19
75	4 : 42	3 : 55	3 : 23	2 : 57
70	5 : 7	4 : 22	3 : 47	3 : 19
65	5 : 48	4 : 57	4 : 17	3 : 46
60	6 : 25	5 : 29	4 : 45	4 : 11
55	7 : 4	6 : 3	5 : 14	4 : 37
50	7 : 44	6 : 37	5 : 44	5 : 3
45	8 : 26	7 : 13	6 : 15	5 : 31
40	9 : 11	7 : 52	6 : 49	5 : 59
35	10 : 4	8 : 37	7 : 29	6 : 34
30	11 : 3	9 : 27	8 : 13	7 : 13
25	12 : 15	10 : 31	9 : 7	8 : 1
20	13 : 48	11 : 50	10 : 18	9 : 4
15	15 : 56	13 : 44	11 : 57	10 : 32
10	19 : 26	16 : 46	14 : 38	12 : 55
5	6 : 36	23 : 9	20 : 20	18 : 1

TABLE II.

Which ſhews the Value of $\frac{\text{Sin. }\varphi^2}{\text{Sin. }\psi}$ *for each of the ſeven Principal Species of Veſſel, when the Obliquity of the Sails, or the Angle* AFS $= \eta$ *is given.*

Angle	Species.		
AFS $= \eta$	$a = 3\ b$	$a = 3\frac{1}{2}\ b$	$a = 4\ b$
90°	0,0741	0,0466	0,0312
85	0,0738	0,0464	0,0311
80	0,0741	0,0470	0,0314
75	0,0751	0,0477	0,0320
70	0,0766	0,0489	0,0328
65	0,0781	0,0503	0,0339
60	0,0820	0,0524	0,0354
55	0,0864	0,0550	0,0373
50	0,0918	0,0585	0,0398
45	0,0972	0,0628	0,0426
40	0,1059	0,0688	0,0468
35	0,1109	0,0753	0,0515
30	0,1314	0,0863	0,0593
25	0,1468	0,0965	0,0704
20	0,1800	0,1209	0,0842
15	0,2269	0,1535	0,1080
10	0,3004	0,2124	0,1515
5	0,4602	0,4493	0,2642

TABLE II.

Which ſhews the Value of $\frac{\text{Sin. } \phi^2}{\text{Sin. } \psi}$ *for each of the ſeven principal Species of Veſſel, when the Obliquity of the Sails or the Angle* AFS$=\eta$ *is given.*

Angle AFS$=\eta$	Species. $a = 4\frac{1}{2}\, b$	$a = 5\, b$	$a = 5\frac{1}{2}\, b$	$a = 6\, b$
90°	0,0219	0,0160	0,0120	0,0092
85	0,0219	0,0161	0,0122	0,0094
80	0,0221	0,0162	0,0123	0,0095
75	0,0226	0,0165	0,0124	0,0096
70	0,0233	0,0170	0,0127	0,0097
65	0,0239	0,0174	0,0131	0,0101
60	0,0250	0,0182	0,0137	0,0106
55	0,0262	0,0191	0,0145	0,0113
50	0,0281	0,0207	0,0155	0,0121
45	0,0298	0,0221	0,0167	0,0129
40	0,0332	0,0244	0,0184	0,0142
35	0,0367	0,0273	0,0203	0,0159
30	0,0424	0,0311	0,0236	0,0182
25	0,0499	0,0374	0,0264	0,0207
20	0,0605	0,0447	0,0340	0,0269
15	0,0780	0,0582	0,0444	0,0346
10	0,1124	0,0845	0,0647	0,0508
5	0,2012	0,1551	0,1209	0,0960

§ 30.

By means of theſe two Tables we are able to determine the Courſe of a Ship for every Obliquity A F S $= \eta$ of its Sails, and for each Species of Veſſel when the Wind V F acts upon the Aft Part of the Sails S B s: For the firſt Table will indicate the Leeway, or the Angle A F X $= \varphi$, and the ſecond will give the Value of Expreſſion $\frac{\text{Sin}\,.\,\varphi^2}{\text{Sin}\,.\,\psi}$, which we ſhall now ſuppoſe $= s$; therefore if the Velocity of the Veſſel be denoted by v, the Reſiſtance will be equal to

$$\frac{v^2}{4g}\cdot\frac{3}{4}ae\,.\,s$$

Plate II. Fig. 8. Now let the Right Line V F repreſent the Direction of the true Wind, the Velocity of which is ſuppoſed $= c$, and the Obliquity of Incidence, or the Angle V F S $= \theta$; it will then be firſt neceſſary to find the apparent Wind by the Rule heretofore given; but ſince this Reſearch might be embarraſſing, we ſhall here lay down a Method much more ſimple. Having ſuppoſed the Line V F $= c$, let us take upon the Direction F X of the Veſſel, the Portion F $x = v$, then if T F Z be drawn perpendicular to the Sail S F s, and from the Points V and X, we let fall the Perpendiculars V T and x Z;

the Line T F will repreſent the Velocity of the Wind in a Direction perpendicular to that of the Sail, and the Line F Z the Velocity with which the Sail recedes directly from the Wind: Hence it follows that the Sail will be ſtruck in the ſame Manner as if it was at Reſt, and received the direct Action of the Wind with a Velocity $= TF - FZ$. Now becauſe the Angle $SFV = \theta$, and the Angle $SFx = \eta + \varphi$, we ſhall have

$$FT = c\,.\,\text{Sin.}\,\theta,\ \text{and}\ FZ = v\,.\,\text{Sin.}\,\overline{\eta + \varphi}$$

and conſequently the Velocity of the Wind in a Direction perpendicular to that of the Sail will be equal to

$$c\,.\,\text{Sin.}\,\theta - v\,\text{Sin.}\,\overline{\eta + \varphi}.$$

Therefore by ſuppoſing the Surface of the Sails $f^2 = \alpha b^2$, (See § 25.) the Force of the Wind will be expreſſed by

$$\frac{1}{800}\,.\,\frac{\overline{c\,.\,\text{Sin.}\,\theta - v\,.\,\text{Sin.}\,\overline{\eta + \varphi}}^{\,2}}{4g}\,.\,\alpha b^2.$$

which being equal to the Reſiſtance will give this Equation.

$$v^2\,.\,\frac{3}{4}aes = \frac{1}{800}\,.\,\overline{c\,.\,\text{Sin.}\,\theta - v\,.\,\text{Sin.}\,\overline{\eta + \varphi}}^{\,2}\,.\,\alpha b^2.$$

from whence we find

$$v = \frac{c\,.\,\text{Sin.}\,\theta\,.\,b\sqrt{\alpha}}{\sqrt{600\,.\,aes} + \text{Sin.}\,\overline{\eta + \varphi}\,.\,b\sqrt{\alpha}},\ \text{or}$$

$v =$

CHAP. IV.

$$v = \frac{c \text{ Sin. } \theta}{\text{Sin. } \overline{\eta + \varphi} + \sqrt{\frac{600 \,.\, a e s}{\alpha b^2}}}.$$

This Expreſſion will therefore give the true Velocity with which the Veſſel will run in its Courſe F X.

§ 31.

But ſince we are unable to diſtinguiſh the true Wind when a Veſſel is under Way, and as all our Obſervations then only makes the apparent Wind known to us, or that which immediately enters into the Expreſſion for the acting Force: the Reduction therefore which we have given in the preceding Article, will in part become uſeleſs, and the Velocity of the Veſſel may be determined in a much more eaſy Manner. For if the Line V F $= c$ ſhould now denote the Velocity of the apparent Wind, whoſe Angle of Incidence V F S is ſuppoſed $= \theta$, the acting Force of the Wind will then be expreſſed by

$$\frac{1}{800} \,.\, \frac{c^2 \,.\, \text{Sin. } \theta^2}{4g} \,.\, \alpha b^2,$$

which being made equal to the Reſiſtance $\frac{v^2}{4g} \,.\, \frac{3}{4} a e s$, we ſhall from thence determine the Velocity of the Veſſel, or

$$v = c\,b\,.\,\text{Sin.}\,\theta\,.\,\sqrt{\frac{\alpha}{600\,.\,a\,e\,s}}\,.$$

For Example ſuppoſe a Veſſel of the 5th Species where $a = 5\,b$, and the Depth of Water $e = \frac{1}{3}\,b$, then $a\,e$ will be $= \frac{5}{3}\,b^2$; ſuppoſe alſo that the Sum of the Sails in Action or $f^2 = 4\,b^2$, or rather $\alpha = 4$; finally let the Obliquity of the Sails A F S $= 50° = \eta$, then from the 1ſt Table we ſhall have the Leeway $\varphi = 6° : 37'$, and from the Second

$$\frac{\text{Sin.}\ \varphi^2}{\text{Sin.}\ \psi} = s = 0,\ 0207\,.$$

Let us now ſuppoſe the apparent Velocity of the Wind $= c$, and the Angle of Incidence $= \theta$, then the Velocity of the Veſſel will be expreſſed by

$$v = c\,.\,\text{Sin.}\,\theta\,.\,\sqrt{\frac{4}{600\,.\,\frac{5}{3}\,.\,0{,}0207}}\,,\ \text{or}$$

$$v = c\,.\,\text{Sin.}\,\theta\,.\,\sqrt{\frac{40}{207}} = 0{,}4396\,.\,c\,.\,\text{Sin.}\,\theta\,.$$

Thus if the Obliquity of the Wind V F S was $= 30°$ the Velocity of the Veſſel would be $= 0{,}2198\,.\,c$, and therefore the Velocity of the Veſſel would be nearly the fifth Part of the apparent Wind c.

§ 32.

§ 32.

If we compare the Direction of the Wind V F, with the Courſe of the Veſſel F X, it is at firſt evident that the Angle V F X muſt be greater than the Angle S F X $= \nu + \varphi$. But if we conſider the firſt Table we ſhall ſee that the Sum of the two Angles $\nu + \varphi$, in deſcending, becomes leſs and leſs to a certain Point, beyond which this Sum begins again to increaſe. It is is therefore very important to know the Diſpoſition of the Sails, where the Sum of $\nu + \varphi$ or the Angle S F X will be the leaſt, ſince by giving to the Obliquity of the Wind V F S $= \theta$ the leaſt Value in which the Wind can ſtill act upon the Sail, we ſhall have the Caſe where the Angle V F X will be the leaſt; and therefore in this Caſe the Courſe of the Veſſel F X will approach the Direction of the Wind F V as near as poſſible, or rather the Veſſel will advance to Windward as much as poſſible: The Veſſel is then ſaid to go Cloſe Hauled, and we regard this Quality of lying near the Wind as a very excellent Property in Veſſels; it depends principally upon the Species of Veſſel, and we have already remarked, that in Proportion as the Length of a Veſſel exceeds the Breadth, the better it is calculated to Ply to Windward, or to go Cloſe Hauled. Therefore in order to elucidate this Matter we ſhall here inſert the following Table.

A TABLE *ſhewing the Angles* AFS $= \eta$, *and* AFX $= \varphi$ *for each Species of Veſſel, when their Sum* SFX $= \eta + \varphi$ *is the leaſt poſſible.*

Species of Veſſel.	AFS $= \eta$	AFX $= \varphi$.	SFX $= \eta + \varphi$.
$a = 3\ b$	13°: 7′	29°: 30′	42°: 37′
$a = 3\frac{1}{2} b$	11 : 4	26 : 4	37 : 8
$a = 4\ b$	9 : 54	23 : 45	33 : 39
$a = 4\frac{1}{2} b$	9 : 24	20 : 0	29 : 24
$a = 5\ b$	8 : 2	18 : 27	26 : 29
$a = 5\frac{1}{2} b$	7 : 54	16 : 18	24 : 12
$a = 6\ b$	7 : 10	15 : 4	22 : 14

Theſe laſt Angles S F X being increaſed by that of the Obliquity of the Wind V F S $= \theta$, will ſhew under what Angle each Species of Veſſel is capable of going Cloſe Hauled. But it is obvious we muſt not take the Angle θ too ſmall, ſince the Wind could not then act upon the Sails, becauſe of their Curvature; and for this Reaſon it ſeems that we cannot diminiſh

CHAP. IV. diminiſh the Angle θ beyond one Point, or $11^\circ : 15'$. Therefore by ſuppoſing the Angle V F S = $11^\circ : 15'$, we ſhall have the Angle of Cloſe Hauling V F X, as will be ſeen in the following Table.

A TABLE *ſhewing the Angles of Cloſe Hauling* V F X *for each Species of Veſſel.*

Species of Veſſel.	Angle V F X of Cloſe Hauling.
$a = 3\ b$	$53^\circ : 52'$
$a = 3\frac{1}{2} b$	48 : 23
$a = 4\ b$	44 : 54
$a = 4\frac{1}{2} b$	40 : 39
$a = 5\ b$	37 : 44
$a = 5\frac{1}{2} b$	35 : 27
$a = 6\ b$	33 : 29

From whence we ſee that a Veſſel of the laſt Species $a = 6\ b$ would be capable of lying within three Points of the Wind nearly.

§ 33.

But in theſe Courſes as the Obliquity of the Sails would be very great, or the Angle A F S ſo ſmall that we could ſcarce carry it into Execution,

cution, becauſe of the Curvature of the Sails; it is in general more advantageous not to give to the Angle S F X $= \eta + \varphi$ its leaſt Value, but to take it ſome Degrees greater; in order that the Angle A F S may not be ſo extremely ſmall. For to facilitate this Choice we ſhall ſtill add the following Table.

A TABLE

TABLE III.

Shewing the Angles SFX = $n + \varphi$ *for every Obliquity of the Sails* AFS = n *in the Seven Species of Veſſel.*

Angle	Species of Veſſel.			
AFS = n.	$a = 3b$	$a = 3\frac{1}{2}b$	$a = 4b$	$a = 4\frac{1}{2}b$
90°	90° : 0′	90° : 0′	90° : 0′	90° : 0′
85	89 : 35	88 : 40	87 : 59	87 : 30
80	86 : 31	85 : 11	84 : 14	83 : 33
75	82 : 58	81 : 40	80 : 27	79 : 42
70	79 : 19	77 : 25	76 : 5	75 : 7
65	75 : 30	73 : 24	71 : 54	70 : 48
60	71 : 41	69 : 19	67 : 39	66 : 25
55	67 : 50	65 : 15	63 : 25	62 : 4
50	64 : 1	61 : 11	59 : 12	57 : 44
45	60 : 12	57 : 12	55 : 2	53 : 26
40	56 : 33	53 : 16	50 : 55	49 : 11
35	52 : 26	49 : 30	46 : 57	45 : 4
30	49 : 43	45 : 52	43 : 6	41 : 3
25	46 : 43	42 : 31	39 : 29	37 : 15
20	44 : 17	39 : 42	36 : 20	33 : 48
15	42 : 44	37 : 39	33 : 51	30 : 56
10	42 : 57	37 : 13	32 : 50	29 : 26
5	47 : 37	41 : 9	35 : 52	31 : 36

TABLE III.

Shewing the Angles $SFX = \eta + \varphi$ *for every Obliquity of the Sails* $AFS = \eta$ *in the Seven Species of Veſſel.*

Angle.	Species of Veſſel.		
$AFS = \eta$	$a = 5b$	$a = 5\frac{1}{2}b$	$a = 6b$
90°	90°: 0′	90°: 0′	90°: 0′
85	87 : 9	86 : 52	86 : 37
80	83 : 2	82 : 38	82 : 19
75	78 : 55	78 : 23	77 : 57
70	74 : 22	73 : 47	73 : 19
65	69 : 57	69 : 17	68 : 46
60	65 : 29	64 : 45	64 : 11
55	61 : 3	60 : 14	59 : 37
50	56 : 37	55 : 44	55 : 3
45	52 : 13	51 : 15	50 : 31
40	47 : 52	46 : 49	45 : 59
35	43 : 37	42 : 29	41 : 34
30	39 : 27	38 : 13	37 : 13
25	35 : 31	34 : 7	33 : 1
20	31 : 50	30 : 18	29 : 4
15	28 : 44	26 : 57	25 : 32
10	26 : 46	24 : 38	22 : 55
5	28 : 9	25 : 20	23 : 1

From hence we ſee that by taking the Angle $\eta = 15^\circ$ we ſhall loſe in the Angle of Cloſe Hauling a few Minutes only in the firſt Species, and but two Degrees in that of the laſt; which Difference would not be ſenſible in Practice.

Mr. *Euler* being informed that his *Théorie complette de la Conſtruction et de la Manœuvre de Vaiſſeaux* was Tranſlating into Engliſh, has taken the trouble of communicating the following Elucidations and neceſſary Additions to that Work, which at the Time of Publication did not occur to him.

In ſearching the Proportion between the Angle of the Obliquity of the Courſe φ, and that of the acting Force ψ, we have found (§ 35, Book II) this Equality, $\text{Tang.}\ \psi = \frac{a^3}{2\,b^3} \,.\, \text{Tang.}\ \varphi^2$, which by ſuppoſing $\frac{2\,b^3}{a^3} = \text{Tang.}\ \alpha$ will be changed into $\text{Tang.}\ \alpha \,.\, \text{Tang}\,.\ \psi = \text{Tang.}\ \varphi^2$, and it being propoſed to find in what Caſe the Difference of the Angles ψ and φ will become the greateſt; we ſee this will happen when $2\ \text{Sin.}\ 2\,\psi = \text{Sin.}\ 2\,\varphi$, or $2\ \text{Sin.}\ \psi \,.\, \text{Coſ.}\ \psi = \text{Sin.}\ \varphi \,.\, \text{Coſ.}\ \varphi$. By combining this Equation with that propoſed, or $\text{Tang.}\ \alpha \,.\, \text{Tang.}\ \psi = \text{Tang.}\ \varphi^2$, we ſhall obtain the following one.

$$\text{Coſ.}\,\alpha^2 \,.\, \text{Sin.}\,\varphi^4 - 2\,\text{Sin.}\,\alpha \,.\, \text{Coſ.}\,\alpha \,.\, \text{Sin.}\,\varphi \,.\, \text{Coſ.}\,\varphi + \text{Sin.}\,\alpha^2 \,.\, \text{Coſ.}\,\varphi$$

which agrees perfectly with the Equation of the fourth Degree, found at § 31 Book II. the Solution of which appeared more difficult at firſt View than it really is, as the following Inveſtigation will evince.

$$\text{For Sin. }\varphi^4\text{ is }= \frac{1}{4}\,.\,\overline{1-\text{Coſ. }2\,\varphi}^{\,2}\,;$$

$$\text{and Coſ. }\varphi^4 = \frac{1}{4}\,.\,\overline{1+\text{Coſ. }2\,\varphi}^{\,2}\,.\text{ alſo}$$

$$\text{Sin. }\alpha\,.\,\text{Coſ. }\alpha\,.\,\text{Sin. }\varphi\,.\,\text{Coſ. }\varphi = \frac{1}{4}\,\text{Sin. }2\,\alpha\,.\,\text{Sin. }2\,\varphi.$$

by ſubſtituting theſe for their reſpective Values we ſhall have this Equation.

$$\overline{1-\text{Coſ. }2\,\varphi}^{\,2}\,.\,\text{Coſ. }\alpha^2+\overline{1+\text{Coſ. }2\,\varphi}^{\,2}\,.\,\text{Sin. }\alpha^2-2\,\text{Sin. }2\,\alpha\,.\,\text{Sin. }2\,\varphi=0$$

which will be reduced to

$$\text{Coſ. }2\,\varphi^2-2\,\text{Coſ. }2\,\varphi\,.\,\text{Coſ. }2\,\alpha-2\,\text{Sin. }2\,\alpha\,.\,\text{Sin. }2\,\varphi+1=0$$

and by putting $2\,\varphi=\zeta$ and $2\,\alpha=\beta$ the Equation will be more commodiouſly expreſſed by

$$\text{Coſ. }\zeta^2-2\,\text{Coſ. }\zeta\,.\,\text{Coſ. }\beta-2\,\text{Sin. }\beta\,.\,\text{Sin. }\zeta+1=0$$

In order to reſolve this Equation we ſhall ſuppoſe it to be compoſed of theſe two following ones

$$\text{Coſ. }\zeta+m\,.\,\text{Sin. }\zeta+n=0,\text{ and}$$

$$\text{Coſ. }\zeta-m\,.\,\text{Sin. }\zeta+n'=0,$$

from whence by multiplying them together we have

$$\text{Coſ. }\zeta^2-m^2\,.\,\text{Sin. }\zeta^2+\overline{n+n'}\,.\,\text{Coſ. }\zeta+m\,.\,\overline{n'-n}\,.\,\text{Sin.. }\zeta+nn'=0$$

or rather

$$\overline{1+m^2}\,.\,\text{Coſ. }\zeta^2+\overline{n+n'}\,.\,\text{Coſ. }\zeta+m\,.\,\overline{n'-n}\,.\,\text{Sin. }\zeta+nn'-m^2=0.$$

In order that we may compare this Equation with that which we have to reſolve, it is neceſſary to multiply the propoſed Equation by $1+m^2$, from whence we ſhall have

$$\overline{1+m^2}.\text{Coſ.}\zeta^2 - 2.\overline{1+m^2}.\text{Coſ.}\zeta.\text{Coſ.}\beta - 2.\overline{1+m^2}.\text{Sin.}\beta.\text{Sin.}\zeta + 1 + m^2 =$$

Hence by comparing the homologous Terms we ſhall have theſe Equalities

$$n' + n = -2.\overline{1+m^2}.\text{Coſ.}\beta;\ n' - n = -\frac{2.\overline{1+m^2}}{m}.\text{Sin.}\beta,$$

and $n\,n' = 1 + 2\,m^2$; from the Firſt and Second we obtain

$$n' = -\overline{1+m^2}.\ \overline{\text{Coſ.}\beta + \frac{\text{Sin.}\beta}{m}} \text{ and } n = -\overline{1+m^2}.\ \overline{\text{Coſ.}\beta - \frac{\text{Sin.}}{m}}$$

$$\text{therefore } n\,n' = \overline{1+m^2}^2.\ \overline{\text{Coſ.}\beta^2 - \frac{\text{Sin.}\beta^2}{m^2}} = 1 + 2\,m$$

Now by ſubſtituting μ for m^2 our Equation will be

$$\overline{1+\mu}^2.\ \overline{\text{Coſ.}\beta^2 - \frac{\text{Sin.}\beta^2}{\mu}} = 1 + 2\,\mu, \text{or } \overline{1+\mu}^2.\ \mu.\overline{\text{Coſ.}\beta^2 - \text{Sin.}\beta^2} = \mu.1+$$

which may ſtill be changed into this

$$\overline{1+\mu}^3.\text{Sin.}\beta^2 - \mu.\overline{1+\mu}^2 = -\mu.\overline{1+2\,\mu}. \text{ becauſe Coſ.}\beta^2 = 1 - \text{Sin.}$$

from which Equation we have $\text{Sin.}\beta^2 = \frac{\mu^3}{\overline{1+\mu}^3}$

and $\frac{\mu}{1+\mu} = \text{Sin.}\beta^{\frac{2}{3}}$, conſequently

$$\mu = \frac{\text{Sin.}\beta^{\frac{2}{3}}}{1 - \text{Sin.}\beta^{\frac{2}{3}}} \text{ and } m = \frac{\text{Sin.}\beta^{\frac{1}{3}}}{\sqrt{1 - \text{Sin.}\beta^{\frac{2}{3}}}}.$$

Let us now ſuppoſe that we had ſought the Angle γ ſuch that $\text{Sin.}\gamma^3$ was $= \text{Sin.}\beta$; then

$$m \text{ will be} = \frac{\text{Sin.}\gamma}{\sqrt{1 - \text{Sin.}\gamma^2}} = \text{Tang.}\gamma,$$

and from thence we obtain

$$1 + m^2 = \frac{1}{\text{Coſ.}\gamma^2};\ \text{Coſ.}\beta + \frac{\text{Sin.}\beta}{m} = \frac{\text{Sin.}\overline{\beta+\gamma}}{\text{Sin.}\gamma};$$

$$\text{and Coſ. }\beta - \frac{\text{Sin. }\beta}{m} = \frac{\text{Sin. }\overline{\gamma-\beta}}{\text{Sin. }\gamma},\text{ therefore}$$

$$n' = -\overline{1+m^2}\,.\,\overline{\text{Coſ.}\beta + \frac{\text{Sin. }\beta}{m}} = -\frac{\text{Sin.}\overline{\beta+\gamma}}{\text{Sin.}\gamma\,.\,\text{Coſ.}\gamma^2};$$

$$n = -\overline{1+m^2}\,.\,\overline{\text{Coſ.}\beta - \frac{\text{Sin.}\beta}{m}} = -\frac{\text{Sin.}\overline{\gamma-\beta}}{\text{Sin.}\gamma\,.\,\text{Coſ.}\gamma^2}.$$

hence our two Equations

$$\text{Coſ. }\zeta + m\,.\,\text{Sin. }\zeta + n = 0,\text{ and}$$
$$\text{Coſ. }\zeta - m\,.\,\text{Sin. }\zeta + n' = 0,$$

will be expreſſed in this Manner

$$\text{Coſ.}\zeta + \text{Tang.}\gamma\,.\,\text{Sin.}\zeta - \frac{\text{Sin.}\overline{\gamma-\beta}}{\text{Sin.}\gamma\,.\,\text{Coſ.}\gamma^2} = 0,\text{ and}$$

$$\text{Coſ.}\zeta - \text{Tang.}\gamma\,.\,\text{Sin.}\zeta - \frac{\text{Sin.}\overline{\beta+\gamma}}{\text{Sin.}\gamma\,.\,\text{Coſ.}\gamma^2} = 0,\text{ or rather}$$

$$\text{Coſ.}\overline{\zeta-\gamma} = \frac{\text{Sin.}\overline{\gamma-\beta}}{\text{Sin.}\gamma\,.\,\text{Coſ.}\gamma}\text{ and Coſ.}\overline{\zeta+\gamma} = \frac{\text{Sin.}\overline{\beta+\gamma}}{\text{Sin.}\gamma\,.\,\text{Coſ.}\gamma}$$

but as theſe two Equations cannot at the ſame Time furniſh real Values for the Angle ζ, ſince it would then follow, that this Angle might have four real Values; it is therefore neceſſary to examine which of theſe two Equations will give two real Values for the Angle ζ; with reſpect to the Firſt, it is evident, that it will furniſh real Values, becauſe the Sin. γ . Coſ. γ, is greater than Sin. $\overline{\gamma-\beta}$; that is to ſay, that the Sin. γ . Coſ. γ is greater than Sin. γ . Coſ. β — Coſ. γ . Sin. β. But as we have ſuppoſed Sin. $\gamma^3 =$ Sin. β, the Coſ. β will be $= \sqrt{1-\text{Sin.}\gamma^6} = \text{Coſ.}\gamma\,.\,\sqrt{1+\text{Sin.}\gamma^2+\text{Sin.}\gamma^4}$, conſequently the Sin. $\overline{\gamma-\beta} =$ Sin. γ . Coſ. β — Sin. β . Coſ. γ, will be equal to

CHAP. IV.

$\text{Sin.}\,\gamma\,.\,\text{Coſ.}\,\gamma\,.\,\overline{\sqrt{\overline{1 + \text{Sin.}\,\gamma^{2} + \text{Sin.}\,\gamma^{4}}} - \text{Sin.}\,\gamma^{2}}$ but it is evident, that $1 + \text{Sin.}\,\gamma^{2}$, or $\sqrt{1 + 2\,\text{Sin.}\,\gamma^{2} + \text{Sin.}\,\gamma^{4}}$ is greater than $\sqrt{1 + \text{Sin.}\,\gamma^{2} + \text{Sin.}\,\gamma^{4}}$; hence it follows that $\text{Sin.}\,\gamma\,.\,\text{Coſ.}\,\gamma$ will be greater than

$$\text{Sin.}\,\gamma\,.\,\text{Coſ.}\,\gamma\,.\,\overline{\sqrt{\overline{1 + \text{Sin.}\,\gamma^{2} + \text{Sin.}\,\gamma^{4}}} - \text{Sin.}\,\gamma^{2}},$$

or rather $\text{Sin.}\,\gamma\,.\,\text{Coſ.}\,\gamma$ greater than $\text{Sin.}\,\overline{\gamma - \beta}$. Thus we are very well aſſured, that the Firſt of our Equations will always give real Values, and that the other will only furniſh imaginary ones. Now there only remains to find two Values of ζ by means of the Equation

$$\text{Coſ.}\,\overline{\zeta - \gamma} = \frac{\text{Sin.}\,\overline{\gamma - \beta}}{\text{Sin.}\,\gamma\,.\,\text{Coſ.}\,\gamma}.$$

For this Purpoſe let us ſuppoſe

$\dfrac{\text{Sin.}\,\overline{\gamma - \beta}}{\text{Sin.}\,\gamma\,.\,\text{Coſ.}\,\gamma} = \text{Coſ.}\,\varepsilon$, and we ſhall have $\zeta - \gamma = \varepsilon$, or rather $\zeta' - \gamma = -\varepsilon$, therefore $\zeta = \varepsilon + \gamma$, and $\zeta' = \gamma - \varepsilon$; hence it follows that $\varphi = \frac{1}{2}\,\zeta$ will be $= \dfrac{\gamma + \varepsilon}{2}$, and $\varphi' = \dfrac{\gamma - \varepsilon}{2}$, conſequently $\varphi + \varphi' = \gamma$, $\varphi - \varphi' = \varepsilon$. Thus all the Operations upon which the Solution of our Problem turns, are reduced to find two Angles γ and ε, the firſt of which is found from the Equation $\text{Sin.}\,\gamma^{3} = \text{Sin.}\,\beta = \text{Sin.}\ 2\,\alpha$, or $\text{Sin.}\,\gamma = \text{Sin.}\ 2\,\alpha^{\frac{1}{3}}$ and the ſecond from the Equation Coſ. $\varepsilon = \frac{\text{Sin.}\ \overline{\gamma - \beta}}{\text{Sin.}\,\gamma\,.\,\text{Coſ.}\,\gamma}$. for having found theſe two Angles the two real Values of φ would be $\varphi = \frac{\gamma + \varepsilon}{2}$ and $\varphi' = \frac{\gamma - \varepsilon}{2}$.

When the Value of φ is determined it will be very eaſy to deduce the Angle ψ from the Equation Tang. α . Tang. ψ = Tang. φ^2, or rather 2 Sin. 2 ψ = Sin. 2 φ. But it is not even neceſſary to have recourſe to theſe two Equations, ſince by means of them we obtain this Equality Sin. 2 φ = Sin. $\overline{2\,\alpha - 4\,\psi}$, the Demonſtration of which is as follows: Becauſe

$$\text{Tang.}\ \varphi^2 = \text{Tang.}\ \alpha\ .\ \text{Tang.}\ \psi, \text{ and}$$

$$\text{Cot.}\ \varphi^2 = \text{Cot.}\ \alpha\ .\ \text{Cot.}\ \psi,$$

the Coſ. φ^2 will be $= \frac{\text{Coſ.}\ \alpha\ .\ \text{Coſ.}\ \psi}{\text{Coſ.}\ \overline{\alpha - \psi}}$, and

$$\text{Sin.}\ \varphi^2 = \frac{\text{Sin.}\ \alpha\ .\ \text{Sin}\ \psi}{\text{Coſ.}\ \overline{\alpha - \psi}}, \text{ therefore}$$

Sin. φ^2. Coſ. φ^2 = 4 Sin. ψ^2. Coſ. ψ^2 will be $= \frac{\text{Sin.}\ \alpha\ .\ \text{Coſ.}\ \alpha\ .\ \text{Sin.}\ \psi\ .\ \text{Coſ.}\ \psi}{\text{Coſ.}\ \overline{\alpha - \psi}^{\,2}}$, and conſequently $2\ \text{Sin.}\ 2\,\psi = \frac{\text{Sin.}\ 2\,\alpha}{2\ \text{Coſ.}\ \overline{\alpha - \psi}^{\,2}} = \frac{\text{Sin.}\ 2\,\alpha}{1 + \text{Coſ.}\ \overline{2\,\alpha - 2\,\psi}}$ from whence we obtain

$$2\ \text{Sin.}\ 2\,\psi + 2\ .\ \text{Sin}\ 2\,\psi\ .\ \text{Coſ.}\ \overline{2\,\alpha - 2\,\psi} = \text{Sin.}\ 2\,\alpha,$$

or $2\ \text{Sin}\ 2\,\psi = \text{Sin.}\ 2\,\alpha - 2\ \text{Sin.}\ 2\,\psi\ .\ \text{Coſ.}\ \overline{2\,\alpha - 2\,\psi}$ but becauſe $\text{Sin.}\ 2\,\alpha - 2\ \text{Sin.}\ 2\,\psi\ .\ \text{Coſ.}\ \overline{2\,\alpha - 2\,\psi} = \text{Sin.}\ \overline{2\,\alpha - 4\,\psi}$ it follows that $2\ \text{Sin.}\ 2\,\psi = \text{Sin.}\ 2\,\varphi$

will be $= \text{Sin.}\ \overline{2\,\alpha - 4\,\psi}$; from this laſt Equation we ſhall have $2\,\alpha - 4\,\psi' = 2\,\varphi'$, or rather $4\,\psi - 2\,\alpha = 360^\circ - 2\,\varphi$, from the firſt Equality we find $\psi' = \frac{1}{2} . \overline{\alpha - \varphi'}$; and from the

Second $\psi = 90^\circ + \frac{1}{2} . \overline{\alpha - \varphi}$.

Although the Solution of the propoſed Problem be very ſimple, it may be ſtill conſiderably abridged in all Caſes where the Fraction $\frac{2\,b^3}{a^3}$ is very ſmall, that is to ſay, when the Angle α is very ſmall, for in this Caſe the leaſt Value of the Angle φ, which we have denoted by φ', will become nearly equal to $\frac{1}{2}\,\alpha$, and the correſponding Value of $\psi' = \frac{1}{4}\,\alpha$; from whence we ſhall find $\varphi = \gamma - \varphi' = \gamma - \frac{1}{2}\,\alpha$, and $\psi = 90^\circ + \frac{3}{4}\,\alpha - \frac{1}{2}\,\gamma$. By making the Application of theſe Rules to the Values of the Fraction $\frac{b}{a}$ which may be in uſe for the various Kinds of Veſſel, the following Table has been calculated.

A TABLE *ſhewing the greateſt Difference between the Obliquity of the acting Force* AFY $= \psi$, *and the Angle of Leeway* AFX $= \varphi$, *and alſo the Angle* SFX $= \eta + \varphi$ *which the Sails make with the Courſe* FX, *for each Species of Veſſel.*

Fraction $\frac{a}{b}$	Angle AFX $= \varphi$.	Angle AFY $= \psi$.	Difference $\psi - \varphi$.	Angle SFX $= \eta + \varphi$.
3,0	29° : 46′	77° : 14′	47° : 28′	42° : 32′
3,1	28 : 50	77 : 30	48 : 40	41 : 20
3,2	27 : 57	77 : 46	49 : 49	40 : 11
3,3	27 : 7	78 : 2	50 : 55	39 : 5
3,4	26 : 21	78 : 17	51 : 56	38 : 4
3,5	25 : 37	78 : 32	52 : 55	37 : 5
3,6	24 : 55	78 : 46	53 : 51	36 : 9
3,7	24 : 15	79 : 0	54 : 45	35 : 15
3,8	23 : 38	79 : 14	55 : 36	34 : 24
3,9	23 : 3	79 : 27	56 : 24	33 : 36
4,0	22 : 29	79 : 39	57 : 10	32 : 50
4,1	21 : 57	79 : 51	57 : 54	32 : 6
4,2	21 : 26	80 : 3	58 : 37	31 : 23
4,3	20 : 56	80 : 15	59 : 19	30 : 41

A TABLE *ſhewing the greateſt Difference between the Obliquity of the acting Force* A F Y $= \psi$, *and the Angle of Leeway* A F X $= \varphi$; *and alſo the Angle* S F X $= \eta + \varphi$ *which the Sails make with the Courſe* F X, *for each Species of Veſſel.*

Fraction $\frac{a}{b}$	Angle AFX $= \varphi$.	Angle AFY $= \psi$.	Difference $\psi - \varphi$	Angle SFX $= \eta + \varphi$.
4,4	20°: 28′	80°: 26′	59°: 58′	30 : 2′
4,5	20 : 1	80 : 37	60 : 36	29 : 24
4,6	19 : 36	80 : 47	61 : 11	28 : 49
4,7	19 : 12	80 : 57	61 : 45	28 : 15
4,8	18 : 48	81 : 7	62 : 19	27 : 41
4,9	18 : 25	81 : 17	62 : 52	27 : 8
5,0	18 : 3	81 : 26	63 : 23	26 : 37
5,1	17 : 42	81 : 35	63 : 53	26 : 7
5,2	17 : 22	81 : 44	64 : 22	25 : 38
5,3	17 : 2	81 : 52	64 : 50	25 : 10
5,4	16 : 44	82 : 0	65 : 16	24 : 44
5,5	16 : 26	82 : 8	65 : 42	24 : 18
5,6	18 : 8	82 : 15	66 : 7	23 : 53
5,7	15 : 51	82 : 23	66 : 32	23 : 28

A TABLE *ſhewing the greateſt Difference between the Obliquity of the acting Force* AFY $= \psi$, *and the Angle of Leeway* AFX $= \varphi$; *and alſo the Angle* SFX $= \eta + \varphi$ *which the Sails make with the Courſe* FX, *for each Species of Veſſel.*

Fraction $\frac{a}{b}$	Angle AFX $=\varphi$.	Angle AFY $=\psi$.	Difference $\psi - \varphi$.	Angle SFX $=\eta+\varphi$.
5,8	15°: 35′	82°: 30′	66°: 55′	23°: 5′
5,9	15 : 20	82 : 37	67 : 17	22 : 43
6,0	15 : 5	82 : 44	67 : 39	22 : 21
6,1	14 : 50	82 : 51	68 : 1	21 : 59
6,2	14 : 36	82 : 57	68 : 21	21 : 39
6,3	14 : 22	83 : 3	68 : 41	21 : 19
6,4	14 : 9	83 : 9	69 : 0	21 : 0
6,5	13 : 56	83 : 15	69 : 19	20 : 41
6,6	13 : 43	83 : 20	69 : 37	20 : 23
6,7	13 : 31	83 : 26	69 : 55	20 : 5
6,8	13 : 19	83 : 31	70 : 12	19 : 48
6,9	13 : 7	83 : 37	70 : 30	19 : 30
7,0	12 : 56	83 : 42	70 : 46	19 : 14

It

CHAP. IV.

It is proper to remark that the Angle $\eta = 90 - \psi$ denotes the Obliquity of the Sails, and that the foregoing Table may be conſidered as a Supplement to thoſe given in § 37, Book II. and in § 32 of this Book.

It will be therefore eaſy to conſtruct by means of this Table, one which expreſſes the Angles of Cloſe Hauling; for we have only to add to the Angle $\varphi + \eta$ juſt found 11°. 15′, in order to have the Angle of Cloſe Hauling, ſuppoſing the Obliquity of the Wind to be one Point; and in general the Angle of Cloſe Hauling for each Species of Veſſel will be found by adding to the Obliquity of the Wind, the Angle $\eta + \varphi$, whoſe Values are exhibited in our Table.

CHAP. V.

Upon the greateſt Velocity of Veſſels, their Courſe and the Direction of the Wind being given.

§ 34.

WE propoſe to give in this Chapter the Reſolution of the Problem beforementioned, in which the greateſt Art of a Navigator conſiſts,

The Courſe of a Veſſel and the Direction of the Wind being given, to find the Diſpoſition both of the Veſſel itſelf, as well as the Sails, in order that it may run upon the propoſed Courſe with the greateſt Velocity poſſible.

We ſhall here ſuppoſe that the apparent Wind is given, ſince the Obſervations made on board a Veſſel will only ſhew us the apparent Wind. Let therefore B F be the Direction of the apparent Wind, and its Velocity $= c$; alſo, ſuppoſe the Line F X to repreſent the Courſe of the Veſſel. Theſe two Directions being given, let us denote the Angle V F X by the Letter δ which is therefore known; and for the unknown Quantities which it is neceſſary Plate II. Fig. 7.

ceſſary to determine, ſuppoſe the Obliquity of the Sails A F S $= \eta$; from whence we find by the firſt Table of the preceding Chapter the Leeway of the Veſſel, or the Angle A F X $= \varphi$; therefore the Angle of Incidence of the Wind V F S $= \theta$ will be $= \delta - \eta - \varphi$, from theſe Elements we have already found the Velocity of the Veſſel (ſee § 31.)

$$v = cb \,.\, \text{Sin.}\, \theta \,.\, \sqrt{\frac{\alpha}{600.\, a e s}}.$$

where the Letter s expreſſes the Quantity

$$\frac{\text{Sin.}\, \varphi^2}{\text{Sin.}\, \psi} = \frac{\text{Sin.}\, \varphi^2}{\text{Coſ.}\, \eta}.$$

becauſe $\eta = 90 - \psi$; conſequently to reſolve our Problem, it is required to find the Value of the Angle η when the Expreſſion

$$\frac{\text{Sin.}\, \theta}{\text{Sin.}\, \varphi} \,.\, \sqrt{\text{Coſ.}\, \eta} = \frac{\text{Sin.}\, \overline{\delta - \eta - \varphi}}{\text{Sin.}\, \varphi} \,.\, \sqrt{\text{Coſ.}\, \eta}$$

will be the greateſt poſſible; where it is neceſſary to remember that the Relation between the Angles η and φ is expreſſed by

$$\text{Coſ.}\, \eta = \frac{a^3}{2 b^2} \,.\, \text{Tang.}\, \varphi^2.$$

§ 35.

By treating the above Expreſſion according to the common Methods for determining the Maxima and Minima we find this final Equality,

$$\text{Tang.}\, \overline{\delta - \eta} = \tfrac{1}{2}\, \text{Tang.}\, \eta \,.\, \frac{2 - \text{Tang.}\, \eta \,.\, \text{Tang.}\, \varphi}{\frac{1}{2} - \text{Tang.}\, \eta \,.\, \text{Tang.}\, \varphi}.$$

from whence it is required to derive the Angles η and φ for each propoſed Angle δ. But ſince this would require Calculations too embarraſſing, it will be better to reverſe the Queſtion, and to ſearch the Angle δ for all the Angles η and φ. In Effect having diſpoſed all theſe Values in a Table ſimilar to that in the preceding Chapter, it will be eaſy to reverſe the Queſtion a ſecond Time, and to aſſign for each Angle δ, the Angles η and φ, which will be the Solution of our Problem.

A TABLE

CHAP. V.

A TABLE *which ſhews for each Angle* AFS $= \eta$, *and for each of the Seven Species of Veſſel; the Angle* VFX $= \delta$ *comprehended between the Direction of the Wind* VF, *and the Courſe of the Veſſel* FX.

Angle	Species of Veſſel.			
AFS $= \eta$	$a = 3\,b$	$a = 3\frac{1}{2}b$	$a = 4\,b$	$a = 4\frac{1}{2}\,b$
90°	180°: 0'	180°: 0'	180°: 0'	180°: 0'
85	178 : 53	176 : 50	175 : 41	175 : 0
80	172 : 13	170 : 12	168 : 59	168 : 11
75	165 : 33	163 : 24	162 : 4	161 : 11
70	158 : 40	156 : 23	154 : 57	153 : 59
65	151 : 38	149 : 13	147 : 40	146 : 37
60	144 : 18	141 : 44	140 : 5	138 : 57
55	136 : 46	134 : 4	132 : 21	131 : 10
50	128 : 43	125 : 53	124 : 5	122 : 51
45	120 : 20	117 : 25	115 : 35	114 : 19
40	111 : 10	108 : 10	106 : 19	105 : 1
35	101 : 28	98 : 29	96 : 41	95 : 25
30	90 : 32	87 : 34	85 : 50	84 : 36
25	78 : 36	75 : 58	74 : 24	73 : 18
20	64 : 55	62 : 37	61 : 13	60 : 16
15	50 : 17	48 : 38	47 : 38	46 : 56
10	33 : 20	32 : 21	31 : 45	31 : 19
5	16 : 19	16 : 0	15 : 46	15 : 37

A TABLE *which ſhews for each Angle* A F S $= n$, *and for each of the Seven Species of Veſſel, the Angle* V F X $= \delta$, *comprehended between the Direction of the Wind* V F, *and the Courſe of the Veſſel* F X.

Angle AFS $= n$.	Species of Veſſel.		
	$a = 5\,b$	$a = 5\frac{1}{2}\,b$	$a = 6\,b$.
90°	180°: 0′	180°: 0′	180°: 0′
85	174 : 33	174 : 13	173 : 56
80	167 : 38	167 : 13	166 : 55
75	160 : 33	160 : 5	159 : 44
70	153 : 16	152 : 44	152 : 19
65	145 : 51	145 : 16	144 : 49
60	138 : 8	137 : 31	137 : 3
55	130 : 18	129 : 39	129 : 10
50	121 : 56	121 : 15	120 : 44
45	113 : 23	112 : 41	112 : 8
40	104 : 5	103 : 22	102 : 56
35	94 : 30	93 : 48	93 : 24
30	83 : 41	83 : 1	82 : 29
25	72 : 27	71 : 54	70 : 25
20	59 : 29	59 : 3	58 : 36
15	46 : 23	46 : 4	45 : 44
10	31 : 1	30 : 49	30 : 36
5	15 : 31	15 : 26	15 : 22

§ 36.

When a Veſſel has firſt been at Reſt, and we have regulated the Diſpoſition for the true Wind; upon the leaſt Communication of Motion to it, the Direction of the Wind will appear to change, although in effect it remains the ſame. We muſt therefore at the ſame Time change the Diſpoſition conformable to the Rules which we have juſt given, in Proportion as the Direction of the Wind appears to vary, and in the ſame Manner as if the Wind had actually changed. But as the Velocity which the Wind is capable of communicating to the Veſſel is ſoon acquired, the whole Change may be made in a very ſhort Time; and therefore we ſhall readily know whether the Diſpoſition, which we have given to the Veſſel and to the Sails, are conformable to our Rules or not? And in the latter Caſe we may eaſily do every thing which is neceſſary. But it is proper to obſerve, that in all Caſes where a *Maximum* or *Minimum* is required, a ſmall Deviation from the Rules preſcribed will produce little or no Change in the Effect, ſo that a Navigator may be always very well ſatisfied when the Diſpoſition which he has made does not differ conſiderably from the Rules given, and therefore we ſhall no longer trouble ourſelves with the difference between the true and apparent Wind.

§ 37.

§ 37.

In order to ſhow a Navigator in what Manner he muſt uſe the Table which we have juſt given, we ſhall give an Example for a Veſſel which belongs to the Species $a = 4\frac{1}{2} b$; and we ſhall ſuppoſe that the Direction of the Wind V F, with the Courſe F X which the Veſſel muſt follow, makes an Angle V F X = 138°. 57′; then our Table will ſhew us that it is neceſſary to make the Obliquity of the Sails or the Angle A F S = 60°. But the firſt Table of the preceding Chapter will, in this Caſe, give us the Leeway A F X = 6° : 25′, and therefore the Angle S F X = 66° : 25′, from whence we find the Angle $VFS = \theta = \delta - \eta - \varphi = 72° : 32′$, conſequently the Sails muſt be diſpoſed in ſuch a Manner that their Obliquity, or the Angle A F S, may be = 60°; and afterwards the Direction of the Veſſel muſt be ſuch that the Wind may fall upon the Sails under an Angle V F S = 72° : 32′; it is then certain that the Veſſel will run upon the propoſed Courſe F X with the Leeway A F X = 6° : 25, and with the greateſt Velocity poſſible. We may alſo aſſign the greateſt Velocity itſelf which will in this Caſe be communicated to the Veſſel; for we have only to draw from the ſecond Table of the preceding Chapter the Value of the Expreſſion $\frac{\text{Sin. } \varphi^2}{\text{Sin. } \psi} = s = 0{,}0250$;

Plate II. Fig. 9.

then

then ſince $a = \frac{9}{2} b$, by taking $e = \frac{2}{5} b$ and $f^2 = 3 b^2$ for the Surface of all the Sails, we ſhall find the Velocity of the Veſſel

$$v = c \,.\, \text{Sin. } 72^\circ : 32' \times \sqrt{\frac{3 b^2}{1080 .\, 0{,}025 b^2}}, \text{ or}$$

$$v = \frac{c}{3} \,.\, \text{Sin. } 72^\circ : 32' = 0{,}318\, c;$$

by which it appears that the Veſſel will acquire ſomething more than three tenths of the Velocity of the Wind. Therefore if the Wind ſhould ſtrike the Sail with a Velocity of 30 Feet in a Second, the Velocity of the Veſſel would be 9 Feet in a Second.

§ 38.

We may be ſurpriſed to find in our laſt Table ſhewing the Angles V F X = δ, Values much leſs than the Limits before aſſigned for the Angle of Cloſe Hauling. For Example, by taking the Angle A F S = n = 5°. this Table gives for the firſt Species of Veſſel where $a = 3 b$, the Angle $\delta = 16^\circ : 19'$, although we have before ſeen that ſuch a Veſſel would not lie nearer the Wind than by an Angle V F X = 53° : 52′, but this Surpriſe will very ſoon vaniſh when we conſider the Conditions under which this ſmall Angle V F X = 16° : 19′ ought to have Place; for taking the Angle S F A = n = 5°, the Leeway A F X = φ will be = 42° : 37′, and

Plate II. Fig. 7.

and therefore the Angle S F X $= \eta + \varphi =$ 47° : 37′, which being ſubtracted from the Angle V F X $=$ 16° : 19′ will leave for the Angle V F S the Value $\theta = -$ 31° : 28′, which being Negative ſhews that the Wind muſt ſtrike the Sail on the oppoſite Side, ſo that in this Caſe the Veſſel would have Stern-way. And in Effect ſince the Sine of the Angle θ is Negative, our Expreſſion will likewiſe give a Negative Velocity; which, although it may be a *Maximum* or *Minimum* for that Courſe, muſt be entirely excluded from our Reſearches. In general whenever the Table gives for the Angle V F X $= \delta$ a leſs Value than S F X $= \eta + \varphi$, ſuch Value muſt be rejected as abſolutely impoſſible, ſince the Angle of Incidence of the Wind V F S $= \theta = \delta - \eta - \varphi$ would then become Negative. From this Circumſtance it appears that the leaſt Values which can be admitted for the Angle V F X $= \delta$ are thoſe where $\delta = \eta + \varphi$, or when the Angle $\theta = 0$, in which Caſe the Veſſel would remain at Reſt. We ſhall therefore ſhew theſe true Limits of the Angle V F X $= \delta$ for each Species of Veſſel in the following Table.

A TABLE *of Limits for the Angles* δ, η *and* φ.

Species.	$VFX = \delta$	$SFA = \eta$	$AFX = \varphi$
$a = 3\,b$	42° : 37′	13° : 7′	29° : 30′
$\alpha = 3\frac{1}{2}\,b$	37 : 8	11 : 4	26 : 4
$a = 4\,b$	33 : 39	9 : 54	23 : 45
$a = 4\frac{1}{2}\,b$	29 : 24	9 : 24	20 : 0
$a = 5\,b$	26 : 29	8 : 2	18 : 27
$a = 5\frac{1}{2}\,b$	24 : 12	7 : 54	16 : 18
$a = 6\,b$	22 : 14	7 : 10	15 : 4

§ 39.

In order to ſpare Navigators the Trouble of making Calculations for each particular Caſe, we ſhall here add particular Tables for each Species of Veſſel. The firſt Column of each will contain the given Angles V F X, compriſed between the Direction of the Wind, and the Courſe of the Veſſel. We begin by the leaſt Value of this Angle, when the Veſſel will remain entirely at Reſt, and from thence we ſhall increaſe ſucceſſively unto 180 Degrees: The Second will ſhew the Obliquity of the

Sails, or the Angle A F S $= \eta$; the Third will denote the Leeway, or the Angle A F X $= \varphi$; and the fourth Column will indicate the Sum of the two laſt Angles or S F X $= \eta + \varphi$, which being taken from the Angle δ will leave the Angle of Incidence of the Wind, or the Angle V F S $= \theta = \delta - \eta - \varphi$, expreſſed in the fifth Column: Finally in the ſixth Column we ſhall add the Values of the Formula.

$$s = \frac{\text{Sin. } \varphi^2}{\text{Sin. } \psi} = \frac{\text{Sin. } \varphi^2}{\text{Coſ. } \eta}.$$

which are taken from the ſecond Table of the preceding Chapter, and whereof we ſhall have Need in order to find the Velocity of the Veſſel itſelf.

A TABLE *of Limits for the Angles* δ, η *and* φ.

Species.	$VFX = \delta$	$SFA = \eta$	$AFX = \varphi$
$a = 3\,b$	42° : 37′	13° : 7′	29° : 30′
$\alpha = 3\frac{1}{2}\,b$	37 : 8	11 : 4	26 : 4
$a = 4\,b$	33 : 39	9 : 54	23 : 45
$a = 4\frac{1}{2}\,b$	29 : 24	9 : 24	20 : 0
$a = 5\,b$	26 : 29	8 : 2	18 : 27
$a = 5\frac{1}{2}\,b$	24 : 12	7 : 54	16 : 18
$a = 6\,b$	22 : 14	7 : 10	15 : 4

§ 39.

In order to ſpare Navigators the Trouble of making Calculations for each particular Caſe, we ſhall here add particular Tables for each Species of Veſſel. The firſt Column of each will contain the given Angles V F X, compriſed between the Direction of the Wind, and the Courſe of the Veſſel. We begin by the leaſt Value of this Angle, when the Veſſel will remain entirely at Reſt, and from thence we ſhall increaſe ſucceſſively unto 180 Degrees: The Second will ſhew the Obliquity of the

Sails, or the Angle A F S $= \eta$; the Third will denote the Leeway, or the Angle A F X $= \varphi$; and the fourth Column will indicate the Sum of the two laſt Angles or S F X $= \eta + \varphi$, which being taken from the Angle δ will leave the Angle of Incidence of the Wind, or the Angle V F S $= \theta = \delta - \eta - \varphi$, expreſſed in the fifth Column: Finally in the ſixth Column we ſhall add the Values of the Formula.

$$s = \frac{\text{Sin. } \varphi^2}{\text{Sin. } \psi} = \frac{\text{Sin. } \varphi^2}{\text{Coſ. } \eta}.$$

which are taken from the ſecond Table of the preceding Chapter, and whereof we ſhall have Need in order to find the Velocity of the Veſſel itſelf.

TABLE I.

Shewing the greateſt Velocity of a Veſſel, and the moſt advantageous Method of Setting the Sails, the Direction of the Wind VF *and Courſe* FX *being given, for the firſt Species of Veſſel where* $a = 3b$.

VFX δ	AFS η	AFX φ	SFX $\eta + \varphi$	VFS θ	$s = \frac{\text{Sin. } \varphi^2}{\text{Sin. } \psi}$.
42°: 37′	13°: 7′	29°: 30′	42°: 37′	0°: 0′	— — —
50 : 17	15 : 0	27 : 44	42 : 44	7 : 33	0,2269
64 : 55	20 : 0	24 : 17	44 : 17	20 : 38	0,1800
78 : 36	25 : 0	21 : 43	46 : 43	31 : 53	0,1468
90 : 32	30 : 0	19 : 43	49 : 43	40 : 49	0,1314
101 : 28	35 : 0	17 : 26	52 : 26	49 : 2	0,1109
111 : 10	40 : 0	16 : 33	56 : 33	54 : 37	0,1059
120 : 20	45 : 0	15 : 12	60 : 12	60 : 8	0,0972
128 : 43	50 : 0	14 : 1	64 : 1	64 : 42	0,0918
136 : 46	55 : 0	12 : 50	67 : 50	68 : 56	0,0864
144 : 48	60 : 0	11 : 41	71 : 41	73 : 7	0,0820
151 : 38	65 : 0	10 : 30	75 : 30	76 : 8	0,0781
158 : 40	70 : 0	9 : 19	79 : 19	79 : 21	0,0766
165 : 33	75 : 0	7 : 58	82 : 58	82 : 35	0,0751
172 : 13	80 : 0	6 : 31	86 : 31	85 : 42	0,0741
178 : 53	85 : 0	4 : 35	89 : 35	89 : 18	0,0738
180 : 0	90 : 0	0 : 0	90 : 0	90 : 0	0,0741

TABLE II.

Shewing the greateſt Velocity of a Veſſel, and the moſt advantageous Method of Setting the Sails, the Direction of the Wind VF *and Courſe* FX *being given, for the ſecond Species of Veſſel where* $a = 3\frac{1}{2}b$.

VFX δ	AFS η	AFX φ	SFX $\eta+\varphi$	VFS θ	$s = \frac{\text{Sin. }\varphi^2}{\text{Sin. }\psi^3}$
37°: 8′	11°: 4′	26°: 4′	37°: 8′	0°: 0′	— — —
48 : 38	15 : 0	22 : 39	37 : 39	10 : 59	0,1535
62 : 37	20 : 0	19 : 42	39 : 42	22 : 55	0,1209
75 : 58	25 : 0	17 : 31	42 : 31	33 : 27	0,0965
87 : 34	30 : 0	15 : 52	45 : 52	41 : 42	0,0863
98 : 29	35 : 0	14 : 30	49 : 30	48 : 59	0,0753
108 : 10	40 : 0	13 : 16	53 : 16	54 : 54	0,0688
117 : 25	45 : 0	12 : 12	57 : 12	60 : 13	0,0628
125 : 53	50 : 0	11 : 11	61 : 11	64 : 42	0,0585
134 : 4	55 : 0	10 : 15	65 : 15	68 : 49	0,0550
141 : 44	60 : 0	9 : 19	69 : 19	72 : 25	0,0524
149 : 13	65 : 0	8 : 24	73 : 24	75 : 49	0,0503
156 : 23	70 : 0	7 : 25	77 : 25	78 : 58	0,0489
163 : 24	75 : 0	6 : 40	81 : 40	81 : 44	0,0477
170 : 12	80 : 0	5 : 11	85 : 11	85 : 1	0,0470
176 : 50	85 : 0	3 : 40	88 : 40	88 : 10	0,0464
180 : 0	90 : 0	0 : 0	90 : 0	90 : 0	0,0466

TABLE III.

Shewing the greateſt Velocity of a Veſſel, and the moſt advantageous Method of Setting the Sails, the Direction of the Wind V F *and the Courſe* F X *being given, for the third Species of Veſſel where* $a = 4\,b$.

V F X δ	A F S η	A F X φ	S F X $\eta + \varphi$	V F S θ	$s = \frac{\text{Sin. } \varphi^2}{\text{Sin. } \psi}$.
33° : 39′	9° : 54′	23° : 45′	33° : 39′	0° : 0′	— — —
47 : 38	15 : 0	18 : 51	33 : 51	13 : 47	0,1080
61 : 13	20 : 0	16 : 20	36 : 20	24 : 53	0,0842
74 : 24	25 : 0	14 : 29	39 : 29	34 : 55	0,0704
85 : 50	30 : 0	13 : 6	43 : 6	42 : 44	0,0593
96 : 41	35 : 0	11 : 57	46 : 57	49 : 44	0,0515
106 : 19	40 : 0	10 : 55	50 : 55	55 : 24	0,0468
115 : 35	45 : 0	10 : 2	55 : 2	60 : 33	0,0426
124 : 5	50 : 0	9 : 12	59 : 12	64 : 53	0,0398
132 : 21	55 : 0	8 : 25	63 : 25	68 : 56	0,0373
140 : 5	60 : 0	7 : 39	67 : 39	72 : 26	0,0354
147 : 40	65 : 0	6 : 54	71 : 54	75 : 46	0,0339
154 : 57	70 : 0	6 : 5	76 : 5	78 : 52	0,0328
162 : 4	75 : 0	5 : 27	80 : 27	81 : 37	0,0320
168 : 59	80 : 0	4 : 14	84 : 14	84 : 45	0,0314
175 : 41	85 : 0	2 : 59	87 : 59	87 : 42	0,0311
180 : 0	90 : 0	0 : 0	90 : 0	90 : 0	0,0312

TABLE IV.

Shewing the greateſt Velocity of a Veſſel, and the moſt advantageous Method of Setting the Sails, the Direction of the Wind V F *and Courſe* F X *being given, for the fourth Species of Veſſel where* $a = 4\frac{1}{2} b$.

VFX	AFS	AFX	SFX	VFS	$s = \frac{\text{Sin. } \varphi^2}{\text{Sin. } \psi}$.
δ	η	φ	$\eta + \varphi$	θ	
29°: 24	9°: 24′	20°: 0′	29°: 24′	0°: 0′	— — —
31 : 19	10 : 0	19 : 26	29 : 26	1 : 53	0,1124
46 : 56	15 : 0	15 : 56	30 : 56	16 : 0	0,0780
60 : 16	20 : 0	13 : 48	33 : 48	26 : 28	0,0605
73 : 18	25 : 0	12 : 15	37 : 15	36 : 3	0,0499
84 : 36	30 : 0	11 : 3	41 : 3	43 : 33	0,0424
95 : 25	35 : 0	10 : 4	45 : 4	50 : 21	0,0367
105 : 1	40 : 0	9 : 11	49 : 11	55 : 50	0,0332
114 : 19	45 : 0	8 : 26	53 : 26	60 : 53	0,0298
122 : 51	50 : 0	7 : 44	57 : 44	65 : 7	0,0281
131 : 10	55 : 0	7 : 4	62 : 4	69 : 6	0,0262
138 : 57	60 : 0	6 : 25	66 : 25	72 : 32	0,0250
146 : 37	65 : 0	5 : 48	70 : 48	75 : 49	0,0239
153 : 59	70 : 0	5 : 7	75 : 7	78 : 52	0,0233
161 : 11	75 : 0	4 : 42	79 : 42	81 : 29	0,0226
168 : 11	80 : 0	3 : 33	83 : 33	84 : 38	0,0221
175 : 0	85 : 0	2 : 30	87 : 30	87 : 30	0,0219
180 : 0	90 : 0	0 : 0	90 : 0	90 : 0	0,0219

TABLE V.

Shewing the greateſt Velocity of a Veſſel, and the moſt advantageous Method of Setting the Sails, the Direction of the Wind V F *and Courſe* F X *being given, for the fifth Species of Veſſel where a = 5 b.*

V F X δ	A F S n	A F X φ	S F X $n + \varphi$	V F S θ	$s = \frac{\text{Sin. } \theta^2}{\text{Sin. } \psi}$.
26°: 29′	8°: 2′	18°: 27′	26°: 29′	0°: 0′	——
31 : 1	10 : 0	16 : 46	26 : 46	4 : 15	0,0845
46 : 23	15 : 0	13 : 44	28 : 44	17 : 39	0,0582
59 : 29	20 : 0	11 : 50	31 : 50	27 : 39	0,0447
72 : 27	25 : 0	10 : 31	35 : 31	36 : 56	0,0374
83 : 41	30 : 0	9 : 27	39 : 27	44 : 14	0,0311
94 : 38	35 : 0	8 : 37	43 : 37	51 : 1	0,0273
104 : 5	40 : 0	7 : 52	47 : 52	56 : 13	0,0244
113 : 23	45 : 0	7 : 13	52 : 13	59 : 10	0,0221
121 : 56	50 : 0	6 : 37	56 : 37	65 : 19	0,0207
130 : 18	55 : 0	6 : 3	61 : 3	69 : 15	0,0191
138 : 8	60 : 0	5 : 29	65 : 29	72 : 39	0,0182
145 : 51	65 : 0	4 : 57	69 : 57	75 : 54	0,0174
153 : 16	70 : 0	4 : 22	74 : 22	78 : 54	0,0170
160 : 33	75 : 0	3 : 55	78 : 55	81 : 38	0,0165
167 : 38	80 : 0	3 : 2	83 : 2	84 : 36	0,0162
174 : 33	85 : 0	2 : 9	87 : 9	87 : 24	0,0161
180 : 0	90 : 0	0 : 0	90 : 0	90 : 0	0,0160

TABLE VI.

Shewing the greateſt Velocity of a Veſſel, and the moſt advantageous Method of Setting the Sails; the Direction of the Wind V F *and Courſe* F X *being given for the ſixth Species of Veſſel where* $a = 5\frac{1}{2}\,b$.

V F X δ	A F S n	A F X φ	S F X $n+\varphi$	V F S θ	$s = \frac{\text{Sin.}\,\varphi^2}{\text{Sin.}\,\psi}$.
24°: 12′	7°: 54′	16°: 18′	24°: 12′	0°: 0′	——
30: 49	10 : 0	14 : 38	24 : 38	6 : 11	0,0647
46 : 4	15 : 0	11 : 57	26 : 57	19 : 7	0,0444
59 : 3	20 : 0	10 : 18	30 : 18	28 : 45	0,0340
71 : 54	25 : 0	9 : 7	34 : 7	37 : 47	0,0264
83 : 1	30 : 0	8 : 13	38 : 13	44 : 48	0,0236
93 : 48	35 : 0	7 : 29	42 : 29	51 : 19	0,0203
103 : 22	40 : 0	6 : 49	46 : 29	56 : 33	0,0184
112 : 41	45 : 0	6 : 15	51 : 15	61 : 26	0,0167
121 : 15	50 : 0	5 : 44	55 : 44	65 : 31	0,0155
129 : 39	55 : 0	5 : 14	60 : 14	69 : 25	0,0145
137 : 31	60 : 0	4 : 45	64 : 45	72 : 46	0,0137
145 : 16	65 : 0	4 : 17	69 : 17	75 : 59	0,0131
152 : 44	70 : 0	3 : 47	73 : 47	78 : 57	0,0127
160 : 5	75 : 0	3 : 23	78 : 23	81 : 42	0,0124
167 : 13	80 : 0	2 : 38	82 : 38	84 : 35	0,0123
174 : 13	85 : 0	1 : 52	86 : 52	87 : 21	0,0122
180 : 0	90 : 0	0 : 0	90 : 0	90 : 0	0,0120

TABLE VII.

Shewing the greateſt Velocity of a Veſſel, and the moſt advantageous Method of Setting the Sails, the Direction of the Wind V F *and Courſe* F X *being given, for the ſeventh Species of Veſſel where* $a = 6\,b$.

V F X δ	A F S n	A F X φ	S F X $n + \varphi$	V F S θ	$s = \frac{\text{Sin.}\,\varphi^2}{\text{Sin}\,\psi}$.
22°: 14′	7°: 10′	15°: 4′	22°: 14′	0°: 0′	——
30 : 36	10 : 0	12 : 55	22 : 55	7 : 41	0,0508
45 : 44	15 : 0	10 : 32	25 : 32	20 : 12	0,0346
58 : 36	20 : 0	9 : 4	29 : 4	29 : 32	0,0269
70 : 25	25 : 0	8 : 1	33 : 1	37 : 24	0,0207
82 : 29	30 : 0	7 : 13	37 : 13	45 : 16	0,0182
93 : 24	35 : 0	6 : 34	41 : 34	51 : 50	0,0159
102 : 56	40 : 0	5 : 59	45 : 59	56 : 57	0,0142
112 : 8	45 : 0	5 : 31	50 : 31	61 : 37	0,0129
120 : 44	50 : 0	5 : 3	55 : 3	65 : 41	0,0121
129 : 10	55 : 0	4 : 37	59 : 37	69 : 33	0,0113
137 : 3	60 : 0	4 : 11	64 : 11	72 : 52	0,0106
144 : 49	65 : 0	3 : 46	68 : 46	76 : 3	0,0101
152 : 19	70 : 0	3 : 19	73 : 19	79 : 0	0,0097
159 : 44	75 : 0	2 : 57	77 : 57	81 : 47	0,0096
166 : 55	80 : 0	2 : 19	82 : 19	84 : 36	0,0095
173 : 56	85 : 0	1 : 37	86 : 37	87 : 19	0,0094
180 : 0	90 : 0	0 : 0	90 : 0	90 : 0	0,0092

§ 40.

Therefore whenever the given Angle V F X $= \delta$ is found either to correſpond exactly, or to be nearly equal to thoſe exhibited in the foregoing Tables, we may then give to the Angles η and φ the Values which are there aſſigned, becauſe a ſmall Difference in the Angle V F S $= \theta$ would not effect the greateſt Velocity. But if this Angle δ ſhould deviate conſiderably from thoſe in the Tables, it will not be difficult to take ſome juſt Mean; this we ſhall explain by an Example. Suppoſe the Veſſel to be of the fifth Species, or $a = 5\, b$, and the propoſed Angle V F X $= \delta = 90°$; then it is evident by taking a Mean between the Angles 83° : 41′ and 94° : 38′ found in the fifth Table, we may put $\eta = 33°$, $\varphi = 8°$ and $s = 0{,}028$; hence it will be neceſſary that we direct the Veſſel with reſpect to the Courſe F X, in ſuch a Manner that the Leeway or the Angle A F X may become $= 8°$; and the Sails muſt be ſo diſpoſed that the Angle A F S may become $= 33°$; then S F X will be $= 41°$, and therefore the Angle of Incidence V F S $= \theta$ will be $= 49°$, which will be the moſt advantageous Diſpoſition. But if the Velocity of the Veſſel itſelf be required, we have

Plate III. Fig. 10.

$$v = c \,.\, \text{Sin.}\, \theta \,.\, \sqrt{\frac{\alpha\, b^2}{600 \,.\, a\, e\, s}},$$

where $\alpha\, b^2$ expreſſes the whole Surface of the

Sails,

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Sails, which we ſhall ſuppoſe to be $= 4\,b^2$, let us alſo ſuppoſe the Depth of the Body $e = \frac{1}{3}\,b$, then becauſe $a = 5\,b$ we ſhall have $a\,e = \frac{5}{3}\,b^2$, and therefore

$$v = c\,.\,\text{Sin.}\,49^\circ\,.\,\sqrt{\frac{1}{7}} = 0{,}2853\,c,$$

by which it appears that the Velocity of the Veſſel will be to that of the Wind as $28\frac{1}{2}$ to 100: Thus, if the Velocity of the Wind was 30 Feet in a Second, that of the Veſſel would be $8\frac{1}{2}$ Feet. This will therefore be the greateſt Velocity which this Veſſel could acquire under the preſcribed Circumſtances.

CHAP. VI.

Upon the beſt Method of Plying to Windward.

§ 41.

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HAVING already ſeen that a Veſſel may run ſuch a Courſe as to make with the Direction of the Wind an Angle more or leſs acute, we readily conceive the Poſſibility of Directing the Courſe of a Veſſel in ſuch a Manner that it may at laſt arrive at a Place directly to Windward. For ſuppoſe F be the Place from whence the Veſſel departs, and that the Wind blows in the Direction V F, then if F″ ſituated immediately to Windward, be the Place which the Veſſel is bound to, it is evident that it would be impoſſible to arrive there by the direct Courſe F F″; but for this Purpoſe it will be firſt neceſſary to run in an oblique Direction; for Example, towards the Right in the Direction F X, which may be executed by an infinite Number of Ways according as the Angle V F X is more or leſs great; and we have ſhewn the Diſpoſitions neceſſary to make the Veſſel run with the greateſt Velocity poſſible in this Courſe F X. Then being arrived at X we change the Diſpoſition of the Veſſel and go upon the other Tack, in order that the Veſſel may now run towards the Left, under the ſame Obliquity as before with reſpect to the Wind, and the Direction of this Courſe X F X′ will be equally inclined to the Direction F V; now being arrived at X′ we may again change the Diſpoſition, ſo that the Veſſel may run upon the Courſe X′ F″ ſtill equally inclined to the Direction of the Wind: From whence we ſee that by ſuch Management the Veſſel may at laſt arrive at the Place propoſed, by making one or more Traverſes as Circumſtances may require. Such a Courſe made by Traverſe Sailing, is called Plying to Windward, by which Mariners are able to avail themſelves of a Wind even directly contrary to the Courſe they wiſh to Steer. We propoſe in this Chapter to ſearch the neceſſary Diſpoſitions, in order that a Veſſel may run from F to F″ directly to Windward, in the ſhorteſt Time poſſible.

§ 42.

It may at firſt be thought that it would be proper to eſtabliſh the Angle V F X as ſmall as poſſible, provided the Veſſel had ſome way through the Water: but it is neceſſary to conſider, that the more we diminiſh this Angle and approach the Limits marked in the ſeven laſt Tables of the preceding Chapter, the leſs will the Velocity of the Veſſel be; by which it appears evidently advantageous to increaſe this Angle, in order to obtain a greater Velocity for the Veſſel, and that ſhe may advance to Windward as quick as poſſible. After this Remark it is obvious, that to effect our Purpoſe it will be neceſſary to take the Angle $VFX = \delta$ ſuch, that not only the Velocity v which will be communicated to the Veſſel, but alſo that which will reſult for the Direction F V ſhould become the greateſt poſſible. This Velocity being $= v \,.\, \text{Coſ.}\, \delta$, it will be neceſſary to ſearch the Diſpoſition when the Value of this Expreſſion $v \,.\, \text{Coſ.}\, \delta$ will be the greateſt poſſible.

§ 43.

Having eſtabliſhed the four Angles η, φ, θ and δ, whoſe Significations have already been ſo often explained, the Velocity of the Veſſel has been found

$$v = cb.\,\text{Sin.}\,\theta.\sqrt{\frac{\alpha}{600.\,aes}} = cb.\,\text{Sin.}\,\theta.\sqrt{\frac{\alpha\,.\,\text{Coſ.}\,\eta}{600.\,ae\,.\,\text{Sin.}\,\varphi^{2}}},$$

It is therefore neceſſary that this Expreſſion, multiplied by the Coſ. δ, ſhould be a *Maximum*; by omitting the known Factor $cb.\sqrt{\frac{\alpha}{600\,.\,ae}}$, it is evident that Sin. θ . Coſ. δ . $\sqrt{\frac{\text{Coſ.}\,\eta}{\text{Sin.}\,\varphi^{2}}}$, or rather $\frac{\text{Sin.}\,\theta\,.\,\text{Coſ.}\,\delta\,.\,\sqrt{\text{Coſ.}\,\eta}}{\text{Sin.}\,\varphi}$ muſt be a *Maximum.* Here it is neceſſary to remark that the Angle φ is already determined by the Angle η; and that the Angle θ being known, we have alſo $\delta = \theta + \eta + \varphi$, ſo that we have here but two indeterminate Angles η and θ, therefore we muſt find theſe two Angles η and θ ſuch that the above Expreſſion may beome the greateſt poſſible. For this Purpoſe let us at firſt ſuppoſe the Angle η to be already known, in which Caſe the Angle φ will alſo be a given Quantity, and therefore the Formula Sin. θ . Coſ. δ muſt be a *Maximum*, becauſe the other Factor $\sqrt{\frac{\text{Coſ.}\,\eta}{\text{Sin.}\,\varphi}}$ has its juſt Value. But we know by Trigonometry that this Product Sin. θ . Coſ. δ is equal to

$$\tfrac{1}{2}\,\text{Sin.}\,\overline{\delta + \theta} - \tfrac{1}{2}\,\text{Sin.}\,\overline{\delta - \theta}\,.$$

therefore ſince $\delta = \eta + \varphi + \theta$, we ſhall have $\delta - \theta = \eta + \varphi$, hence the Sine of this Angle is already determined, and there now only remains to make the Sine of the Angle $\delta + \theta$ as great as poſſible, which will evidently happen when $\delta + \theta = 90°$.

§ 44.

After this Diſcovery we only require the Angle η which may ſatisfy our Queſtion, ſince we have already reſolved this Problem in the preceding Chapter; where we have aſſigned for each Angle δ the three other Angles η, φ and θ, in order that the Velocity of the Veſſel may be the greateſt poſſible for each propoſed Courſe. Here is therefore a very ſimple Solution of the required Problem; we have only to ſearch in the particular Tables, which we have given at the End of the preceding Chapter, the Caſe where the Sum of the two Angles $\delta + \theta$ will give 90°. and then we ſhall have for each Species of Veſſel the juſt Angle δ, this will determine the Courſe F X, which the Veſſel muſt follow in Plying to Windward. Thus, for the firſt Species of Veſſel the Angle $\delta = 64° : 55'$ gives $\delta + \theta = 85° : 33'$, which is too ſmall, and the following Angle $\delta = 78° : 36'$ gives $\delta + \theta = 110° : 29'$ which is too great; from whence we ſee that it is neceſſary to take the Angle $\delta = 67° \frac{1}{2}$, we ſhall then have the Angle $\theta = 22° \frac{1}{2}$, and the correſponding Angles $\eta = 21^{\circ}\frac{1}{4}$ and $\varphi = 23^{\circ}\frac{3}{4}$, conſequently for a Veſſel of the firſt Species we muſt regulate the Courſe for Plying to Windward in the following Manner.

Plate III. Fig. 12.

$$\text{I. } VFX = \delta = 67^{\circ}\tfrac{1}{2};$$

$$\text{II. } AFS = \eta = 21^{\circ}\tfrac{1}{4};$$

$$\text{III. } AFS = \varphi = 23^{\circ}\tfrac{3}{4}, \text{ and}$$

$$\text{IV. } VFS = \theta = 22^{\circ}\tfrac{1}{2}.$$

and for the Velocity we ſhall have $s = 0{,}174$, therefore $600\,s = 104{,}4$, conſequently becauſe $a = 3\,b$ we ſhall have

$$v = c\,.\,\text{Sin.}\,\theta\,.\sqrt{\frac{\alpha\,b}{313{,}2\,.\,e}}$$

hence the Velocity againſt the Wind, or

$$v\,.\,\text{Coſ.}\,\delta = v.\,\text{Sin.}\,\theta = c.\,\text{Sin.}\,\theta^{2}.\sqrt{\frac{\alpha\,b}{313{,}2\,.\,e}},$$

from whence we find this Velocity

$$v\,.\,\text{Coſ.}\,\delta = 0{,}0083\,.\,c\sqrt{\frac{\alpha\,b}{e}},$$

where c denotes the Velocity of the Wind.

§ 45.

By the ſame Means we may calculate for the other Species of Veſſel, by ſearching the Angles $VFX = \delta$, $AFS = \eta$, $AFX = \varphi$, $SFX = \eta + \varphi$, and $VFS = \theta$; and alſo the Velocity againſt the Wind $v\,.\,\text{Coſ.}\,\delta = v\,.\,\text{Sin.}\,\theta$, which

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will be reduced to this Form $N \,.\, c \,.\, \sqrt{\frac{\alpha k}{e}}$, where N expreſſes a certain Decimal Fraction: By making the neceſſary Calculations for each Species of Veſſel we ſhall obtain the following Table.

A TABLE *ſhewing the moſt advantageous Diſpoſition for Plying to Windward.*

Species of Veſſel.	VFX δ	AFS η	AFX φ	V.FS θ	Fraction N
$a = 3\ b$	$67^{\circ}\frac{1}{2}$	21°	$23^{\circ}\frac{3}{4}$	$22^{\circ}\frac{1}{2}$	0,0083
$a = 3\frac{1}{2}\ b$	65	21	$19\frac{1}{4}$	25	0,0114
$a = 4\ b$	$63\frac{1}{4}$	21	16	$26\frac{3}{4}$	0,0142
$a = 4\frac{1}{2}\ b$	62	$20\frac{3}{4}$	$13\frac{1}{4}$	28	0,0177
$a = 5\ b$	$61\frac{1}{4}$	$20\frac{1}{2}$	12	$28\frac{3}{4}$	0,0181
$a = 5\frac{1}{2}\ b$	$60\frac{1}{2}$	$20\frac{1}{2}$	$10\frac{1}{2}$	$29\frac{1}{2}$	0,0232
$a = 6\ b$	$60\frac{1}{4}$	$20\frac{1}{2}$	10	$29\frac{3}{4}$	0,0273

With reſpect to this Table it is neceſſary to remark, that the Calculations are not made to the greateſt Preciſion, becauſe in Practice we can never reckon upon one Degree, ſo that a greater Exactneſs would have been ill-placed; beſides, it would have required very troubleſome Calculations. Thus this Table will be very ſufficient to direct us in Practice.

§ 46.

By conſidering this Table we very ſoon perceive, that all the Art of a Navigator in Plying to Windward is reduced to the two following Rules.

Rule I. Regards the Angle $AFS = \eta$, which the Sails ought to make with the Axe of the Veſſel; and we ſee that this Angle only varies from 21° to $20^\circ\frac{1}{2}$: Since therefore the Difference of half a Degree could no Ways produce any Change in the Effect, a Navigator may always diſpoſe the Sails in ſuch a Manner that their Obliquity, with reſpect to the Axe of the Veſſel, be about 21° or rather $20^\circ\frac{1}{2}$.

Rule II. Reſpects the Angle $VFS = \theta$ under which the Wind ſtrikes the Sails: This Angle varies in our Table from $22^\circ\frac{1}{2}$ to $29^\circ\frac{1}{2}$. Since therefore Veſſels in general appertain to ſome mean Species between $a = 3b$ and $a = 6b$, a Navigator may always take a Mean between theſe two Limits, which will be nearly 26°, ſo that having Set the

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Plate III. Fig. 12.

Sails S F *s* according to the firſt Rule, nothing more is required than to direct the Veſſel in ſuch a Manner that the Wind may ſtrike it under an Angle of 26°. However it will be proper to obſerve if the Veſſel ſhould belong to ſome of the firſt Species, this Angle of 26° muſt be diminiſhed by ſome Degrees; but if the Veſſel ſhould approach ſome of the laſt Species, the ſame Angle muſt be increaſed by ſome Degrees: And it may be neceſſary to add, that no very bad Effect will happen in Practice, even ſhould a Deviation of one Degree or ſomething more from the Rule take place; ſince in all Reſearches which relate to a *Maximum* or *Minimum* a ſmall Deviation is not of any Conſequence. But when theſe Rules are obſerved, a Navigator may be aſſured that his Veſſel will run to Windward as quick as poſſible, although he ſhould not know the Leeway or the Angle A F X; the true Value of which may however be eaſily found by our Table.

It is ſtill proper to remark the great Advantage which long Veſſels have over ſhort ones. For we ſee that whilſt the Expreſſion $\sqrt{\frac{\alpha b}{e}}$ remains the ſame, Veſſels of the firſt Species will only advance to Windward with a Velocity 0,0083, when thoſe of the laſt Species will run at the Rate of 0,0273, which is more than three Times as great; ſo that a Veſſel of the ſeventh Species is capable of advancing to Windward three Times as quick as a Veſſel of the firſt.

§ 47.

If inſtead of Plying directly to Windward it was required to advance in a Direction which ſhould deviate a little from it (for with reſpect to Directions which are very diſtant, Veſſels may run in a direct Courſe); the Method which we have before made uſe of will as readily furniſh us with a Solution of this Caſe. Thus, when the Wind blows in the Direction V F, and when it is required to advance againſt the Direction F U, the Angle V F U being $= \gamma$; let us ſuppoſe as before the Angle V F X $= \delta$, where F X repreſents the Courſe of the Veſſel; and let the Angle under which the Wind ſtrikes the Sails be $= \theta$; alſo let us take a Space $F x = v$, equal to the Velocity of the Veſſel, and from x upon F U let fall the Perpendicular $x n$; then becauſe the Angle U F X $= \delta - \gamma$ it

Plate III. Fig. 13.

it is requiſite that v . Cof. $\overline{\delta - \gamma}$ = Space F u ſhould be a *Maximum.* Therefore conſidering the Angles η and φ as already having their juſt Value, it is neceſſary that this Expreſſion Sin. θ . Cof. $\overline{\delta - \gamma}$ ſhould have the greateſt Value poſſible; which will happen when the Sum of theſe two Angles $\theta + \delta - y$ become a right Angle, from thence we ſhall have $\delta + \theta = 90^\circ + \gamma$. And nothing more will then be required than to ſearch in our Tables the particular Caſe where the Sum of the two Angles $\delta + \theta$ becomes equal to $90 + \gamma$; from thence we ſhall find at the ſame time both the Angles η and φ. But as this Queſtion is rarely to be met with, it would be ſuperfluous to employ any more time upon it, and we ſhall only remark that we muſt not reckon too much upon an exact Agreement between Experience and our Determinations when the Veſſel is Plying to Windward. The Reaſon is very evident, ſince in this Caſe the Wind not only Strikes the Sails, but alſo Acts upon the whole Surface of the Veſſel, as well as the Maſts and Rigging, by which the Motion of the Veſſel may be very much obſtructed; from whence, the Effect produced by the Sails muſt likewiſe be very conſiderably altered, and that ſo much the more as the Wind is more violent; becauſe the Waves of the Sea will then alſo concur to retard the Way of the Veſſel.

CHAP.

CHAP. VII.

Explanations reſpecting the different Species of Veſſel.

§ 48.

WHEN we eſtabliſhed the different Species of Veſſel by the different Proportions between the Length of the Body *a* and its Breadth *b*, we were led thereto by conſidering the Proportion between the Reſiſtance of the Fore-Part, and that of the greateſt tranſverſe Section of the Body, when each are ſuppoſed to be moved in a direct Manner through the Water. But having conſulted ſome Experiments made upon Ships of War, we have ſuppoſed this Proportion as $\frac{2b^2}{a^2+2b^2}$ to Unity; which is an Harmonic Mean between the two extream Caſes, and between which it ſeems we may conſider all Veſſels which are in Uſe. But we ſee evidently, that this Hypotheſis is founded upon a certain Law, by which the Breadth of the Body is diminiſhed from the Middle to the Extremity of the Fore Part, or Stem; ſo that whenever the Conſtruction of a Veſſel deviates from this Law, it might well happen that the Reſiſtance of the Fore Part would become either greater or leſs than our Formula: Conſequently ſince the different Deſtinations of Veſſels require alſo different Configurations for the Fore Part, we muſt not be ſurpriſed when the Application of our Formula often deviates from the Truth. It is therefore to remedy this Defect that we ſhall add here the following Explanations upon the true Character of the different Species of Veſſel.

§ 49.

At firſt it is proper to remark, that in direct Courſes an Error in our Formula would be of no Conſequence, ſeeing that in Navigation it is a Matter of little Import, whether a Veſſel ſails either a little more ſlow or more quick than our Rule indicates, ſince we do not always take the trouble either to meaſure the Velocity of the Wind, or the Surface of the Sails exactly, in order to compare the actual Velocity of the Veſſel with our Determination: And however great this Difference may be, it will not have any Influence upon the Management of Veſſels themſelves. But it is not the ſame in Oblique Courſes where a Knowledge of the Leeway, which makes a very eſſential Part of the Art of conducting Veſſels properly, depends principally on the Truth of our Formula: So that if it ſhould deviate much from the Truth it might occaſion very diſagreeable Conſequences. For having ſup-

poſed

Plate II. Fig. 7.

poſed the Angle A F Y = ψ, which the acting Force F Y makes with the Direction F A of the Veſſel, and the Angle A F X = φ, which the Courſe of the Veſſel F X makes with its proper Direction F A, and which is the Leeway of the Veſſel; we have eſtabliſhed conformable to our Hypotheſis this Relation between the two Angles ψ and φ:

$$\text{Tang. } \psi = \frac{a^3}{2\,b^3} . \text{ Tang. } \varphi^2.$$

From whence we ſee that if this Equality ſhould deviate much from the Truth, the Rules which we have preſcribed for oblique Courſes might be productive of Errors ſufficiently conſiderable. But it is principally upon this Equality that we have eſtabliſhed the different Species of Veſſels, rather than upon the immediate Proportion between the Length and Breadth of the Body. Thus our firſt Species characterized by $a = 3\,b$ muſt actually contain all the Veſſels, where this Relation has place Tang. $\psi = \frac{27}{2}$. Tang. φ^2, even when the Proportion between a and b ſhould be quite different from $\alpha = 3\,b$. In the ſame Manner the true Character of our ſecond Species expreſſed by $a = \frac{7}{2}\,b$ muſt comprehend all the Veſſels where we have Tang. $\psi = \frac{343}{16}$. Tang. φ^2, or nearly Tang. $\psi = 22$. Tang. φ^2 whatever the Proportion between a and b ſhould have; and it is the ſame of all the other Species. It is therefore from thence that we muſt judge to which Specie each propoſed Veſſel appertains: But in order to ſhew this by an Example, let us ſuppoſe for any given Veſſel this Proportion to have place, Tang. $\psi = 50$. Tang. φ^2, we have only to compare the Number 50 with our Coefficient $\frac{a^3}{2b^3}$ and we ſhall have $\frac{a}{b} = \sqrt[3]{100}$; or rather $a = 4\frac{2}{3}b$. From whence we ſee that this Veſſel muſt be ranged between our fourth Species expreſſed by $a = 4\frac{1}{2}b$, and the fifth denoted by $a = 5b$ without regarding the actual Proportion between the Length and Breadth of the Body of the Veſſel.

§ 50.

In order to render this Judgment general, let us ſuppoſe that for any propoſed Veſſel we have found this Relation Tang. $\psi = \mathrm{N}$. Tang. φ^2; and that we put $\frac{a^3}{2b^3} = \mathrm{N}$, from whence we have $\frac{a}{b} = \sqrt[3]{2\,\mathrm{N}}$. Then taking the Cube Root nearly we ſhall ſuppoſe $\sqrt[3]{2\,\mathrm{N}} = n$, and therefore have $a = n\,b$; from whence we ſee, that this Veſſel muſt be related to thoſe of our Species which are expreſſed by $a = n\,b$, although the true Proportion between the Length and Breadth of the Body may be very different from the given one. Moreover it will not be difficult to diſcover the Value of this Number N by a very ſimple Experiment: Suppoſe the Sails to be placed obliquely to the Axe A B, under an Angle A F S $= \eta$, which may be taken at Pleaſure, then directing the Veſſel in ſuch a Manner that the Wind may fall perpendicular upon the Sails, in order to be the better aſſured that the acting Force F Y is alſo perpendicular; in which Caſe we have nothing to fear from the Curvature of the Sails: We ſhall then have the Angle $\psi = 90^\circ - \eta$, and in this Situation, if we let the Veſſel run through ſome Space, and obſerve exactly the true Courſe F X, we ſhall then know the Leeway, or the Angle A F X $= \varphi$. Hence theſe two Angles ψ and φ being determined by this Experiment we ſhall have the Number $N = \frac{\text{Tang. } \psi}{\text{Tang. } \varphi^2}$, and therefore may eaſily determine the true Species to which this Veſſel muſt be related. However, it will be always proper to make theſe Experiments in good Weather and in a Calm Sea, in order that we may have nothing to apprehend from the Agitation of the Waves.

§ 51.

From good Models in Miniature which repreſent Veſſels exactly as they are, very important Experiments upon the Reſiſtance of Veſſels may be very uſefully made; and which is ſo much the more neceſſary, as the Theory upon this Subject is ſtill very defective, as we have already remarked. But for this Purpoſe it is not neceſſary that the Model ſhould repreſent the whole Veſſel entirely; it will be ſufficient to expreſs exactly the Figure of the Body, and in particular its Surface even to a Degree of poliſhing; ſince we have obſerved that a greater or leſs Smoothneſs is capable of producing a ſenſible Change in the Reſiſtance of Veſſels. It would therefore be ſuperfluous to repreſent in ſuch a Model all the interior Parts of the Veſſel, and ſtill leſs all which is above the Water Line, but it is neceſſary to load the Model in ſuch a Manner, that when put into the Water its immerſed Part may correſpond exactly to that of the Veſſel itſelf. We readily conceive that by making ſeveral ſuch ſmall Bodies of different Figures we might obtain very important Concluſions for perfecting the Conſtruction of Veſſels; ſince the Experiments which we might eaſily make, would lead us without any difficulty to a Diſcovery of the good or bad Properties, which great Veſſels executed according to ſuch Models ought to have with Reſpect to the Reſiſtance. As for the other Properties of Veſſels, the Theory is already ſo well eſtabliſhed that we have no further Occaſion to have Recourſe to Experiments.

§ 52.

In order to make theſe Experiments a great Baſon *l l n n* is required, in which the Model may move freely through a ſufficient Space. Let us ſuppoſe therefore, that we are deſirous of diſcovering the Reſiſtance which ſuch a ſmall Veſſel would experience from the Fore Part in a direct Courſe by its Motion through the Water; nothing more is required than to faſten a Thread A M N O to the Model which paſſes over a Pulley O without the Baſon; to this we ſuſpend at the other End a Weight P which by its Gravity will draw the ſmall Veſſel through the Space A M N. It is neceſſary to obſerve that the Thread muſt be fixed to the Model in ſuch a Manner, that during its Motion, the juſt Portion of the Body may remain immerſed in the Water, and that the Motion is made through a right Line A N in the Direction of the Axe B A; which will be readily done

Plate IV. Fig. 14.

after ſome Trials are firſt made. When the Veſſel begins to move, its Motion will at firſt be accelerated, but it will very ſoon acquire one that is uniform. Let us ſuppoſe this uniform Motion to happen after paſſing through the Space A M, and that we mark this Place M in the Baſon by the Line *m n*, then as ſoon as the Veſſel has arrived at this Limit M, we muſt reckon by means of a Pendulum the Number of Seconds which is requiſite for it to run through the Space M N, and whenever we have obſerved this Time we ſhall then be able to aſſign the Reſiſtance which this Veſſel will meet with in direct Courſes.

§ 53.

For this Purpoſe we muſt firſt reduce the Weight P to a Volume of Water equally heavy, then P will become an Extent of three Dimenſions; now ſuppoſe r^2 to be a Plane Surface, which would experience the ſame Reſiſtance as the Veſſel, by moving directly through the Water with the ſame Velocity; then r^2 will expreſs the abſolute Reſiſtance of the Veſſel which we ſearch: alſo ſuppoſe v to denote the Velocity with which the Veſſel moves uniformly through the Space M N $= s$, and let the Number of Seconds obſerved be repreſented by t: Then the Letters s, t and P will expreſs known Quantities, whilſt r and v are unknown ones. Now it is evident the Reſiſtance will be $= \frac{v^2}{4g} . r^2$ which muſt therefore be equal to the Force P, hence we have this Equation

$$\frac{v^2 . r^2}{4g} = P$$

but ſince the Motion is made with the Velocity v through the Space s in the Time t we ſhall have $vt = s$, or $v = \frac{s}{t}$, therefore our Equation will become $\frac{s^2 . r^2}{4g . t^2} = P$; from whence we obtain the abſolute Reſiſtance

$$r^2 = \frac{4g . t^2 . P}{s^2}.$$

expreſſed by known Quantities.

§ 54.

By ſuch Experiments made upon the Models of Veſſels we may alſo diſcover the juſt Proportion between the two Angles ψ and φ, which we have juſt expreſſed by this Formula

$$\text{Tang. } \psi = N . \text{Tang. } \varphi^2.$$

We have only to conſider the firſt Origin of this Equality, which was before derived from the Proportion between the Reſiſtance of the Fore Part and the lateral Reſiſtance, which the Veſſel would experience if the Motion was made in the Direction of the leſſer Axe of the Body: For having denoted the Reſiſtance of the Fore Part by the Letter p, and the lateral Reſiſtance by q we have found this Equality (ſee § 32. Book II.)

$$\text{Tang.}\ \psi = \frac{q}{p}.\ \text{Tang.}\ \varphi^2.$$

But the Experiment which we have juſt deſcribed already diſcovers the abſolute Reſiſtance of the Fore Part $p = r^2$; nothing more therefore is required than to make a ſimilar Experiment in order to obtain the other abſolute Reſiſtance q.

§ 55.

For this Purpoſe the Model muſt again be put into the Baſon in ſuch a Manner, that by drawing it through the Space M N, the Direction of the Motion may remain perpendicular to the great Axe A B; which may be eaſily done by means of two Threads faſtened at A and B, and joined together at C: And the Veſſel is again drawn through the ſame Space M N $= s$ by the ſame Weight P. Let us now ſuppoſe that the lateral Reſiſtance which we ſearch is equal to that of a plane Surface R^2, and that the Motion through the Space M N is made in T Seconds, then we ſhall find, as in the preceding Caſe,

$$R^2 = \frac{4g.T^2.P}{s^2}.$$

This

Plate I. Book 3.

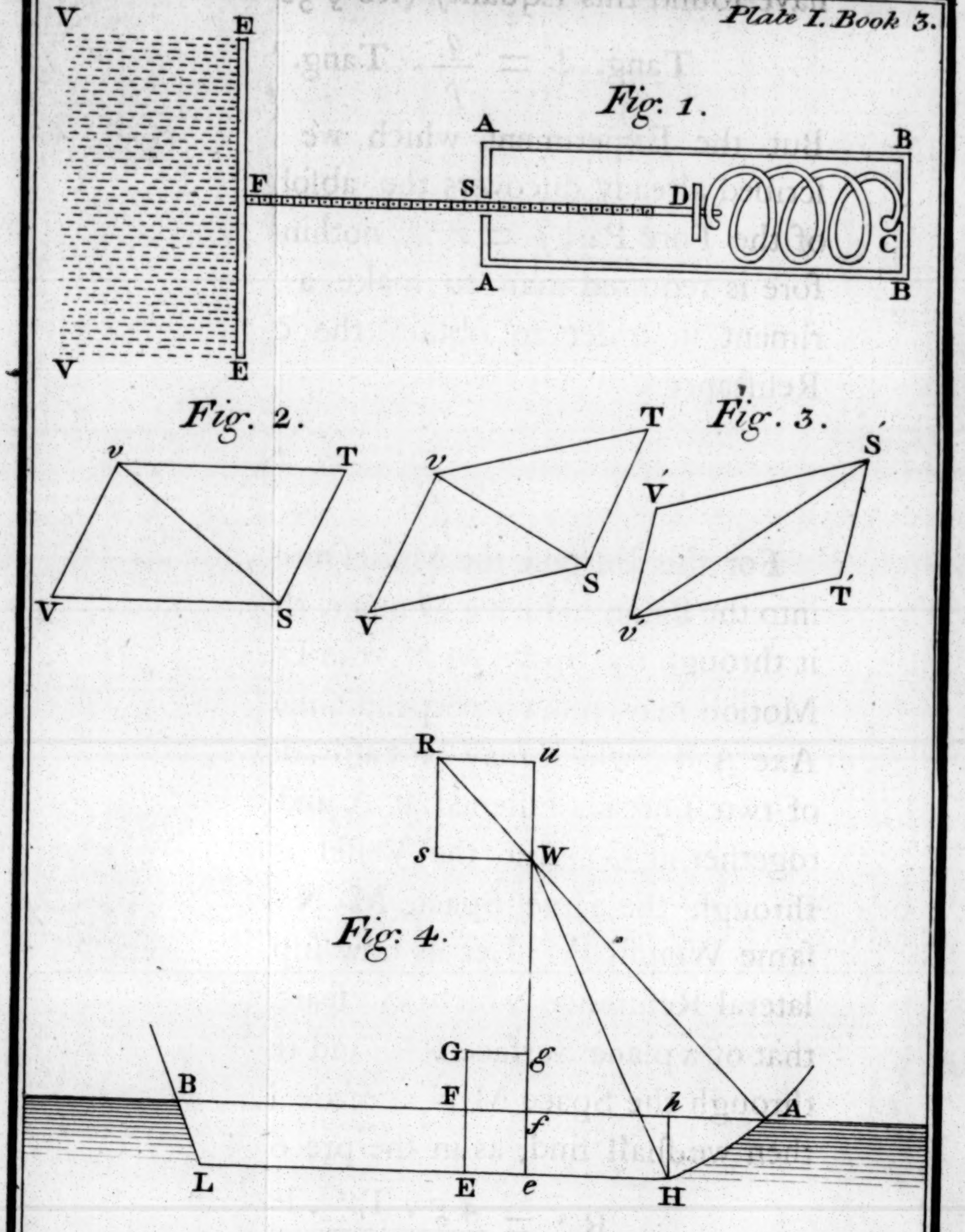


Fig. 5.

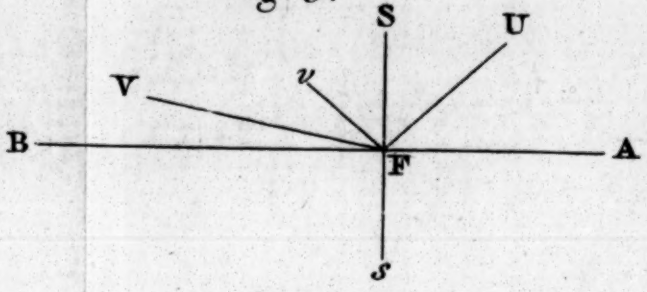


Fig. 6.

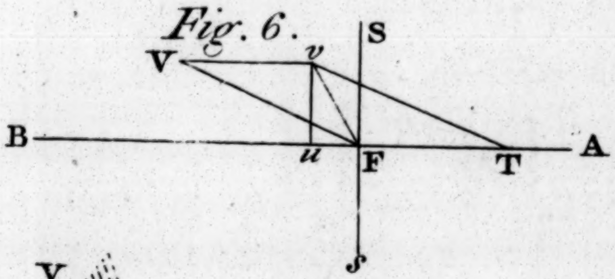
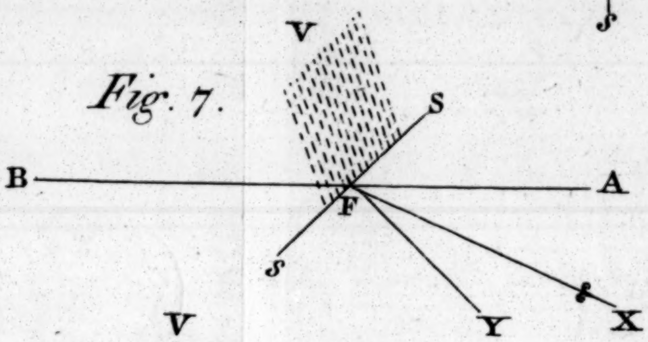


Fig. 7.



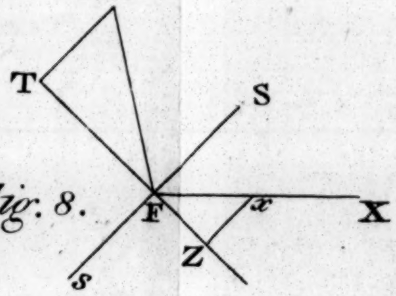
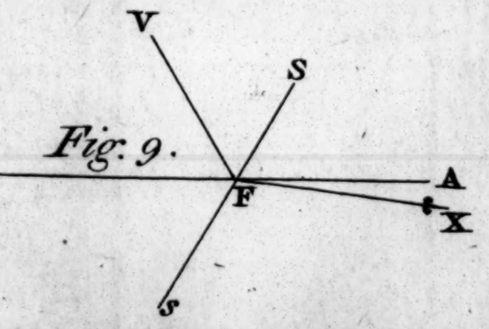


Fig. 8.

Fig. 9.



This Expreſſion will therefore give us the Value of the Letter.

$q = \frac{4g \cdot T^2 \cdot P}{s^2}$; that of p being $= \frac{4g \cdot t^2 \cdot P}{s^2}$.

from whence the Proportion between theſe two Reſiſtances will become $\frac{q}{p} = \frac{R^2}{r^2} = \frac{T^2}{t^2}$; hence this Proportion will be as the Square of the Times T and t, during which the ſame Space M N is run through. Now becauſe

$$N = \frac{\text{Tang. } \psi}{\text{Tang. } \varphi^2} \text{ is alſo } = \frac{q}{p} = \frac{T^2}{t^2}$$

therefore $n = \sqrt[3]{2N}$ will be $= \sqrt[3]{\frac{2T^2}{t^2}}$, and Veſſels conſtructed upon this Model would appertain to the Species which we have expreſſed by $a = n\, b$, whatever may be the true Proportion between the Length a and the Breadth b of the Body.

Let us ſuppoſe, that by an Experiment we had found the two Times $t = 5$ Seconds, and $T = 40$ Seconds, then we ſhould have $N = 64$; and therefore $n = \sqrt[3]{2N} = 5{,}04$. From whence we ſee that ſuch a Veſſel muſt be ranged in our fifth Species $a = 5\, b$ nearly. And it is in general by this Means that we muſt apply the Rules before given for the oblique Courſes of Veſſels.

SUPPLEMENT

UPON THE

ACTION OF OARS.

§ 1.

THE Action of Rowers being one of the moſt laborious, it is without doubt very unfortunate that only one third of their Exertions can be employed to put a Veſſel in Motion, whilſt the other two thirds are in a Manner loſt; either by raiſing the Oars out of the Water, or by moving them through the Air in order to plunge them again into the Water: therefore if the whole Number of Rowers be ſuppoſed $= n$, we can only reckon upon $\frac{1}{3} n$ whoſe continual Exertions are employed to put the Veſſel in Motion: For although in general all the Rowers act at once in ſtriking the Water with their Oars, and continue to repeat their Strokes together, it will nevertheleſs be permitted to ſuppoſe that only $\frac{1}{3} n$ of the Rowers continually act to urge the Veſſel; in order that we may regard the Motion of the Veſſel as Uniform, without being under the neceſſity of giving any attention to the alternate Accelerations and Retardations to which it is in Effect ſubject. Beſides as the Rowers are free Agents, their Action is not ſuſceptible of a more exact Determination, because a Variety of ſufficient Magnitude may even enter into their Efforts without its being almoſt known to themſelves.

§ 2.

Let us at preſent conſider the Force which a Rower is capable of employing; at firſt it is neceſſary to remark, that however great the Efforts may be which a Rower can exert at Reſt, they muſt ſuffer a diminution in Proportion as they are obliged to put their Bodies or their Members in Motion; and there will always be a certain degree of Velocity which they can never ſurpaſs; by which means the more they approach it the more alſo will the Force which they are able to employ become leſs, and will at laſt entirely vaniſh. As it is here of the laſt Importance to take into Conſideration this Variability in the Force of the Rowers, which reſults from their proper Motion; we ſhall therefore endeavour to reduce it to ſome fixed Determination. For this Purpoſe let us ſuppoſe F to be the Force which a Rower at Reſt is capable of employing, and let c be the greateſt Velocity with which he can move his Members and beyond which he is incapable of moving the leaſt Obſtacle: It is now required to aſſign the Force with which this Rower could act when he is obliged to move himſelf with a given Velocity $= u$; but the Formula which expreſſes this Force muſt be ſuch, that taking the Velocity $u = 0$, the Force will become $= F$, and muſt entirely vaniſh when $u = c$.

§ 3.

This diminution of Force is evidently cauſed by the Efforts which the Rowers muſt uſe in their own Perſons; and by which there remains ſo much leſs for them to apply to the Oars. As free Agency itſelf or rather a Caprice may very much influence them, it is without doubt impoſſible to comprize this Variability within any Analytical Expreſſions; but it ſeems that we ſhall not deviate much from the Truth, when we compare this Caſe with the Force of a Current of Water whoſe Velocity is $= c$, and which by ſtriking a Body at Reſt will exert there a Force $= F$. Now if this Body is carried according to the ſame Direction with a Velocity $= u$ leſs than c, we know that the Force of the Current upon this Body will then be

$= F \times \overline{1 - \frac{u}{c}}^{2}$; from whence it alſo ſeems that we may uſe this Formula to expreſs the Force of a Rower for the Caſe propoſed. But it is here proper to obſerve that the Letter F will not ſo much expreſs an abſolute Force, as a Volume of Water whoſe Weight is equal to the Force in Queſtion; and that the Letters *c* and *u*, which we make uſe of to expreſs the Velocities, denote the Spaces run through in one Second of Time with theſe ſame Velocities.

§ 4.

Let us now conſider the Rower in Action, and ſuppoſe *u* to be the Velocity with which his Body and principally his Arms are moved; then the Force which he exerts upon the Oars will be expreſſed by this Formula $F \times \overline{1 - \frac{u}{c}}^{2}$. Therefore ſince the Number of all the Rowers is ſuppoſed $= n$, and that only one third are conſtantly in Action; the Sum of all the Forces which act upon the Oars in order to communicate Motion to the Veſſel will be $= \frac{1}{3} n \,.\, F \times \overline{1 - \frac{u}{c}}^{2}$; and is therefore the total Force which continually acts upon the

Veſſel, this for the Sake of Brevity we ſhall denote by the Letter P, hence

$$P = \frac{1}{3} n . F \times \overline{1 - \frac{u}{c}}^2.$$

§ 5.

Plate IV. Fig. 16. Suppoſe the Line P O Q to repreſent an Oar ſupported at O upon the Side of the Veſſel, and the Part O P within the Veſſel be denoted by p, and the Part O Q without by q; alſo ſuppoſing the Point P to be the Place where the Force of the Rower is applied, and the Point Q the Center of the Oar Blade, with which the Water is ſtruck: Becauſe the Point P of the Oar is drawn by the Rower in the Direction P R with a Velocity $= u$, the Line P R being perpendicular to O P; the other Point Q will receive a Motion according to the Direction Q S with a Velocity $= \frac{q u}{p}$, the Line Q S being in like Manner perpendicular to O Q; ſince we regard the Oar as a Streight Line, or as a Leaver ſupported at the Point O.

§ 6.

Let us now conſider the Motion of the Veſſel itſelf, which moves in the Direction A a, with a Velocity $= v$, then it is evident the Veſſel will experience the ſame Reſiſtance from the Water, which it would ſuffer at Reſt by the Water acting upon it in an oppoſite Direction aA, with a Velocity $= v$; this Conſideration enables us to regard the Veſſel as being effectively at Reſt, and the Efforts of the Rowers as employed to preſerve the Veſſel at Reſt in a Current of Water whoſe Velocity is v. In order to determine the Reſiſtance itſelf, ſuppoſe f^2 to be a Plane Surface, which being ſtruck in a direct Manner by the ſame Current of Water would receive a Shock equal to the Reſiſtance ſought; but we know that the Force of this Reſiſtance will be equal to the Weight of a Maſs of Water whoſe Volume is $= f^2 \cdot \frac{v^2}{4g}$, where g denotes the Height which a heavy Body will fall in one Second of Time. Let us alſo ſuppoſe for Brevity this Force to be denoted by the Letter R, then we ſhall have

$$R = \frac{f^2 \cdot v^2}{4g}.$$

§ 7.

We ſhall now return to the Oar whoſe Point Q has a Motion in the Direction QS with a Velocity $= \frac{qu}{p}$; then ſince the Water itſelf is carried in the Direction aA, with a Velocity $= v$, we ſee that the Oar is not ſtruck by the Water, but in Proportion as its Velocity $\frac{qu}{p}$ exceeds the Velocity v of the Water. But although the Direction Q S be not preciſely the ſame as a A, the Difference can never be very conſiderable; beſides we give to the Oars ſuch a Length, that during the Time of their paſſing through the Water the Direction Q S will differ but little from the Direction of the Veſſel itſelf. Conſequently the Velocity with which the Water is ſtruck by the Oars will be $= \frac{qu}{p} - v$.

§ 8.

Suppoſe now the Surface of the Blade or the Extremity of each Oar to be $= h^2$, and that the Water is ſtruck by it in a perpendicular Manner, or nearly ſo; then the Force which will reſult therefrom will be equal

$$\frac{h^2}{4g} \cdot \overline{\frac{qu}{p} - v}^{\,2},$$

whoſe Direction will be the Right Line Q T, nearly the ſame as that of the Motion; and as this Formula will expreſs the Force of one Rower produced at the Point P of the Oar, the Sum of all theſe Forces will be equal to

$$\frac{1}{3} n \cdot \frac{h^2}{4g} \cdot \overline{\frac{qu}{p} - v}^{\,2},$$

which we ſhall likewiſe for the Sake of Brevity ſuppoſe $=$ Q. Where it is neceſſary to obſerve,

ſerve, that h^2 denotes the Area of the Blade correſponding to one Rower; for if two or more were attached to the ſame Oar, then h^2 would only expreſs the Half or a leſs Part of the whole Surface of the Oar.

§ 9.

Here therefore we have three Forces P, Q, and R, upon which depends the Solution of our Problem, or rather the Determination of the Velocity, which will be communicated to the Veſſel by the *n* Rowers which we ſuppoſe to be attached to it; and firſt ſince we conſider the Veſſel at Reſt, the Nature of the Lever ſuſpended at the Point O, will give us the true Proportion between the Forces P and Q; from whence we have $P . p = Q . q$. But ſince theſe two Forces are only applied to the Oar, we cannot conſider them as immediately acting upon the Body of the Veſſel: For it is principally the Force which the Point O ſuſtains that immediately acts upon the Veſſel; this Force being therefore equal to the Sum $P + Q$, the Point O will be urged thereby in a Direction O A; now, becauſe the Rowers by forcing or drawing the Oars in the Direction P R, ſupport their Bodies and principally their Feet againſt the Veſſel, there will reſult from thence a Force $= P$, and by which the Veſſel will be urged backwards. Hence it is neceſſary to take away this Force from that which is applied to the Point O = P + Q; there will then only remain a Force = Q, by which the Veſſel will be urged in the Direction O A. Conſequently, ſince the Motion of the Veſſel is ſuppoſed Uniform, this Force Q muſt be preciſely equal to the Reſiſtance R; and therefore we have only to reſolve theſe two Equations,

1ſt. $P \,.\, p = Q \,.\, q$, and 2dly. $Q = R$.

§ 10.

The laſt of theſe Equations $Q = R$, by ſubſtituting their ſuppoſed Values will give us

$$\frac{1}{3}\, n \,.\, \frac{h^2}{4g} \,.\, \overline{\frac{qu}{p} - v}^{\,2} = \frac{f^2 v^2}{4g}, \text{ therefore}$$

$$h \,.\, \overline{\frac{qu}{p} - v} \,.\, \sqrt{\frac{1}{3}\, n} = fv, \text{ or rather}$$

$$\frac{qu}{p} - v = \frac{fv}{h \,.\, \sqrt{\frac{1}{3}\, n}}.$$

Again, for the ſake of Brevity, let us ſuppoſe

$$m = \frac{f}{h \,.\, \sqrt{\frac{1}{3}\, n}}, \text{ then we ſhall have}$$

$$f^2 = \frac{1}{3}\, m^2\, h^2\, n, \text{ or } n = \frac{3f^2}{m^2 \,.\, h^2};$$

therefore our Equation will be $\frac{qu}{p} - v = mv$ and conſequently

$\frac{q}{p} =$

$$\frac{q}{p} = \frac{v \, . \, \overline{m+1}}{u} \text{ and } \frac{p}{q} = \frac{u}{v \, . \, \overline{m+1}}.$$

From whence we know the Proportion between the two Parts of each Oar; when the two Velocities v and u and the Number m are given.

§ 11.

Moreover, becauſe $Q = R$ the firſt Equation, $P \, . \, p = Q \, . \, q$ will become

$$P = \frac{q}{p} \, . \, R = \frac{v \, . \, \overline{m+1}}{u} \, . \, R,$$

and by ſubſtituting for P and R their ſuppoſed Values, we ſhall obtain this new Equation,

$$\frac{1}{3} \, n \, . \, F \, . \, \overline{1 - \frac{u}{c}}^2 = \frac{f^2 \, . \, v^3 \, . \, \overline{m+1}}{4g \, . \, u};$$

and therefore

$$\frac{1}{3} \, n \, . \, F \, u \, . \, \overline{1 - \frac{u}{c}}^2 = \frac{f^2 \, . \, v^3 \, . \, \overline{m+1}}{4g}.$$

From which we may find the Velocity v of the Veſſel; the Force of the Rowers with their Number n, their Velocity u, and the Number m being given, ſince we ſhall then have

$$v = \sqrt[3]{\frac{4ng \, . \, F u \, . \, \overline{1 - \frac{u}{c}}^2}{3f^2 \, . \, \overline{m+1}}}.$$

§ 12.

From hence we are able to reſolve this very important Queſtion.

With what Velocity muſt the Rowers act upon the Oars in order to communicate the greateſt Velocity poſſible to the Veſſel?

For we ſee that this Velocity would vaniſh both in the Caſe when $u = 0$, and likewiſe in the Caſe when $u = c$. It is therefore required to find the Value of v, in order that our Formula may obtain the greateſt Value poſſible, which will happen when $u = \frac{1}{3} c$. Here it is very remarkable, that ſince $F \cdot \overline{1 - \frac{u}{c}}^{2}$ expreſſes the Force of one Rower, and u the Velocity with which he acts; the Product $F u \cdot \overline{1 - \frac{u}{c}}^{2}$ will preciſely expreſs that which in Mechanicks we call the Quantity of Action: Hence this Truth merits all our Attention; that the Velocity v communicated to the Veſſel, is proportional to the Cube Root of the Quantity of Action of the Rowers, whilſt the Letter m remains the ſame. But ſince m alſo contains the Number of Rowers, this Proportion will without doubt be more complicated, as we ſhall ſee hereafter: However it is evident that we muſt take $u = \frac{1}{3} c$, which will give the Quantity of

Action $= \frac{4}{27} \cdot F c$, and therefore the greateſt Velocity which the Veſſel can acquire will be expreſſed by

$$\sqrt[3]{\frac{16}{81} \cdot \frac{n g F c}{f^2 \cdot \overline{m+1}}},$$

which by ſubſtituting for n its Value $\frac{3 f^2}{m^2 b^2}$, will become

$$\sqrt[3]{\frac{16}{27} \cdot \frac{g F c}{m^2 b^2 \cdot \overline{m+1}}}:$$

then having found this Velocity each Oar muſt be divided at the Point O in ſuch a Manner that

$$\frac{OQ}{OP} = \frac{p}{q} = \frac{v \cdot \overline{m+1}}{u} = \frac{3 v \cdot \overline{m+1}}{c}.$$

in which Caſe we may be aſſured that the Oars will be diſpoſed in the moſt advantageous Manner, and will communicate to the Veſſel the greateſt Velocity poſſible.

§ 13.

In order to apply theſe Formulas to Practice, we muſt firſt aſſign to F, or the Force which a Man at reſt is able to employ, its juſt Value; and we readily conceive this Force ſhould not be ſuppoſed greater than the Rowers can continue to Exert. By having recourſe to ſome Experiments it appears that we muſt not augment this Force beyond three quarters of a Cubic Foot of Water, or about forty-eight Pounds Averdupoize. Moreover, it ſeems that the greateſt Velocity c which a Man can exert, muſt not ſurpaſs ſeven Feet and a Half; therefore the Value of u or $\frac{1}{3}c$ will become two Feet and a Half. From hence we are able to judge whether a Boat, or any other Veſſel, has its Oars properly diſpoſed or not. For if the Rowers, in drawing the Oars through the Water, move their Arms with a greater Velocity than two Feet and a Half in a Second, it is a very certain Rule that the Work of the Rowers is not well arranged.

§ 14.

Experience has alſo ſhewn us, that the Blade of an Oar uſed by one Man, ſhould never ſurpaſs Half a Square Foot, in order that the Oars may not be too heavy for the Management of the Rowers. Putting therefore $b^2 = \frac{1}{2}$, the Number of Rowers $n = \frac{3f^2}{m^2 b^2}$ will be $= \frac{6f^2}{m^2}$; and by ſuppoſing the Number of Rowers n to be $= \alpha f^2$, the Quantity f^2 being expreſſed in Square Feet, ſince the whole is governed by the Quantity of the Reſiſtance denoted by f^2; we ſhall then have $m^2 = \frac{6}{\alpha}$ and $m = \sqrt{\frac{6}{\alpha}}$. Moreover, ſince the Height $g = 16$ Feet, the two Equations which contain the whole Solution of our Problem will become

$$v = \sqrt[3]{\frac{160.\,\alpha}{9.\,\overline{1+\sqrt{\frac{6}{\alpha}}}}} = \sqrt[3]{\frac{160}{9}\cdot\frac{\alpha}{1+\sqrt{\frac{6}{\alpha}}}},$$

and having found v, the other Equation will be

$$\frac{q}{p} = \frac{3v\,.\,\overline{m+1}}{c} = \frac{2}{5}v\,.\,\overline{1+\sqrt{\frac{6}{\alpha}}}.$$

§ 15.

Theſe Formulas are now very ſufficient to calculate a Table for any Number of Rowers, which will ſhew both the Velocity of the Veſſel or the Space run through in one Second of Time, as well as the Proportion between the two Parts of each Oar, within and without the Veſſel; or rather the Value of $\frac{OQ}{OP}$. The firſt Column in the following Table will repreſent the Number of Rowers, denoted by the Expreſſion αf^2, where f^2 is the abſolute Reſiſtance expreſſed in Square Feet; and for this Purpoſe we ſhall give to the Letter α preciſely the Values of 1, 2, 3, 4, 5, *&c.* to 40, and from thence to 100 by Intervals of Fives and Tens; ſince having examined a Galley with two Hundred and Sixty Rowers we have found the abſolute Reſiſtance $f^2 =$ ten Square Feet, ſo that the Number of Rowers was $26 f^2$. The ſecond Column will ſhew the Velocity of the Veſſel, or the Space run through in one Second; and the third will expreſs the Rate *per* Hour, which is therefore $3600 \,.\, v$; laſtly, the fourth Column will ſhew the juſt Value of $\frac{OQ}{OP}$.

A TABLE *ſhewing the Number of Rowers, the Velocity of the Veſſel, and the Proportion between the two Parts of the Oar within and without the Veſſel.*

Number of Rowers.	Velocity *per* Second.	Rate *per* Hour.	Proportion $\frac{OQ}{OP}$.
1 f^2	1,727	6218	2,383
2 f^2	2,352	8468	2,570
3 f^2	2,806	10080	2,710
4 f^2	3,173	11425	2,824
5 f^2	3,487	12555	2,923
6 f^2	3,764	13551	3,011
7 f^2	4,013	14446	3,091
8 f^2	4,240	15263	3,165
9 f^2	4,449	16017	3,233
10 f^2	4,646	16725	3,290
11 f^2	4,827	17378	3,357
12 f^2	4,999	17998	3,414
13 f^2	5,163	18586	3,468
14 f^2	5,318	19145	3,520
15 f^2	5,466	19679	3,568
16 f^2	5,608	20191	3,617

A TABLE *ſhewing the Number of Rowers, the Velocity of the Veſſel, and the Proportion between the two Parts of the Oar, within and without the Veſſel.*

Number of Rowers.	Velocity *per* Second.	Rate *per* Hour.	Proportion $\frac{OQ}{OP}$.
17 f^2	5,745	20681	3,663
18 f^2	5,876	21153	3,707
19 f^2	6,002	21609	3,750
20 f^2	6,124	22048	3,792
21 f^2	6,243	22474	3,832
22 f^2	6,358	22887	3,871
23 f^2	6,469	23286	3,909
24 f^2	6,577	23676	3,946
25 f^2	6,682	24054	3,982
26 f^2	6,784	24423	4,017
27 f^2	6,884	24782	4,052
28 f^2	6,982	25135	4,086
29 f^2	7,077	25475	4,118
30 f^2	7,170	25810	4,150
31 f^2	7,262	26140	4,181
32 f^2	7,350	26458	4,212

A TABLE *ſhewing the Number of Rowers, the Velocity of the Veſſel, and the Proportion between the two Parts of the Oar, within and without the Veſſel.*

Number of Rowers.	Velocity *per* Second.	Rate *per* Hour.	Proportion $\frac{OQ}{OP}$.
33 f^2	7,437	26773	4,243
34 f^2	7,522	27080	4,273
35 f^2	7,606	27380	4,302
36 f^2	7,688	27678	4,331
37 f^2	7,769	27966	4,359
38 f^2	7,848	28254	4,387
39 f^2	7,926	28533	4,414
40 f^2	8,003	28811	4,441
45 f^2	8,368	30127	4,569
50 f^2	8,708	31347	4,690
60 f^2	9,325	33563	4,908
70 f^2	9,874	35546	5,106
80 f^2	10,374	37347	5,286
90 f^2	10,834	39003	5,452
100 f^2	11,261	40539	5,608

§ 16.

Hence we ſee that Theory does not determine any Thing upon the abſolute Length of the Oars; but if we conſider the Space through which the Rowers move the Point P at one Stroke, and ſuppoſe P R $= r$, it is evident if the Part O P $= p$ was equal to P R $= r$, then the Oar at each Stroke would be moved through an Angle of 60 Degrees, by which Means at the Commencement and End of the Stroke the Direction Q S of the Action upon the Water would be ſo oblique to the Veſſel, that there would reſult a very ſenſible Diminution. Therefore ſince this Space P R may be eſtimated at three Feet nearly, it is neceſſary that the interior Part of the Oar O P be at leaſt five or ſix Feet, and thus the whole Length will now be determined for each Number of Rowers. Finally it is ſtill neceſſary to remark, that if Circumſtances would allow us to give a greater Surface to the Blade than half a ſquare Foot, the Velocity of the Veſſel would be conſiderably increaſed thereby; as may be ſeen by a Table inſerted in the Memoirs of the Academy of Berlin, Vol. III. Page 210, where I have ſuppoſed b^2 to be equal to three-fourths of a ſquare Foot; and we may alſo ſee that the Proportion

Fig. 10.

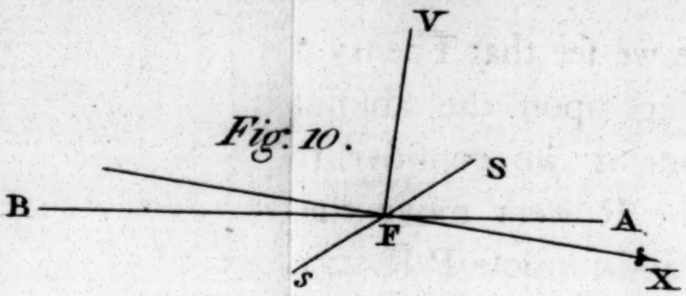


Fig. 11.

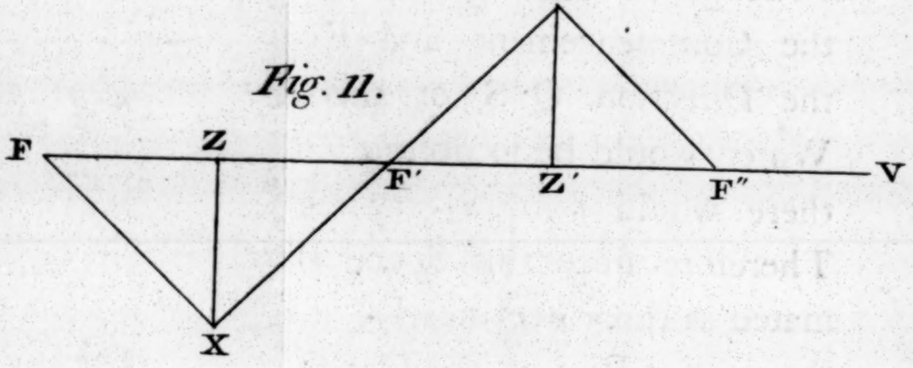


Fig. 12.

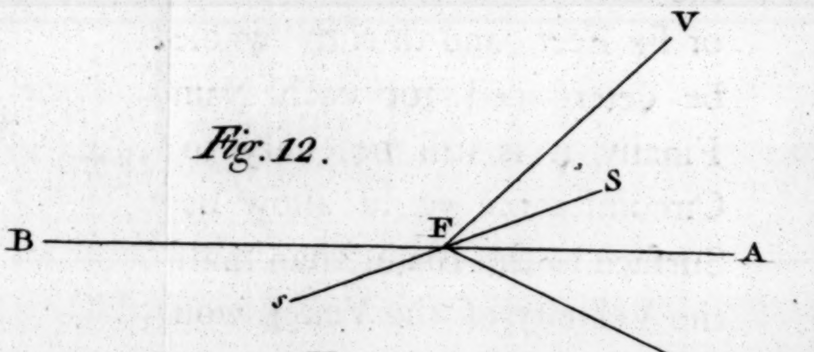


Fig. 13.

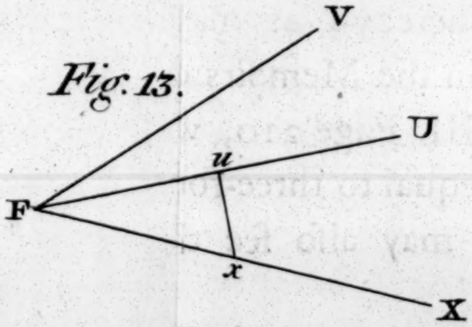


Plate IV. Book 3.

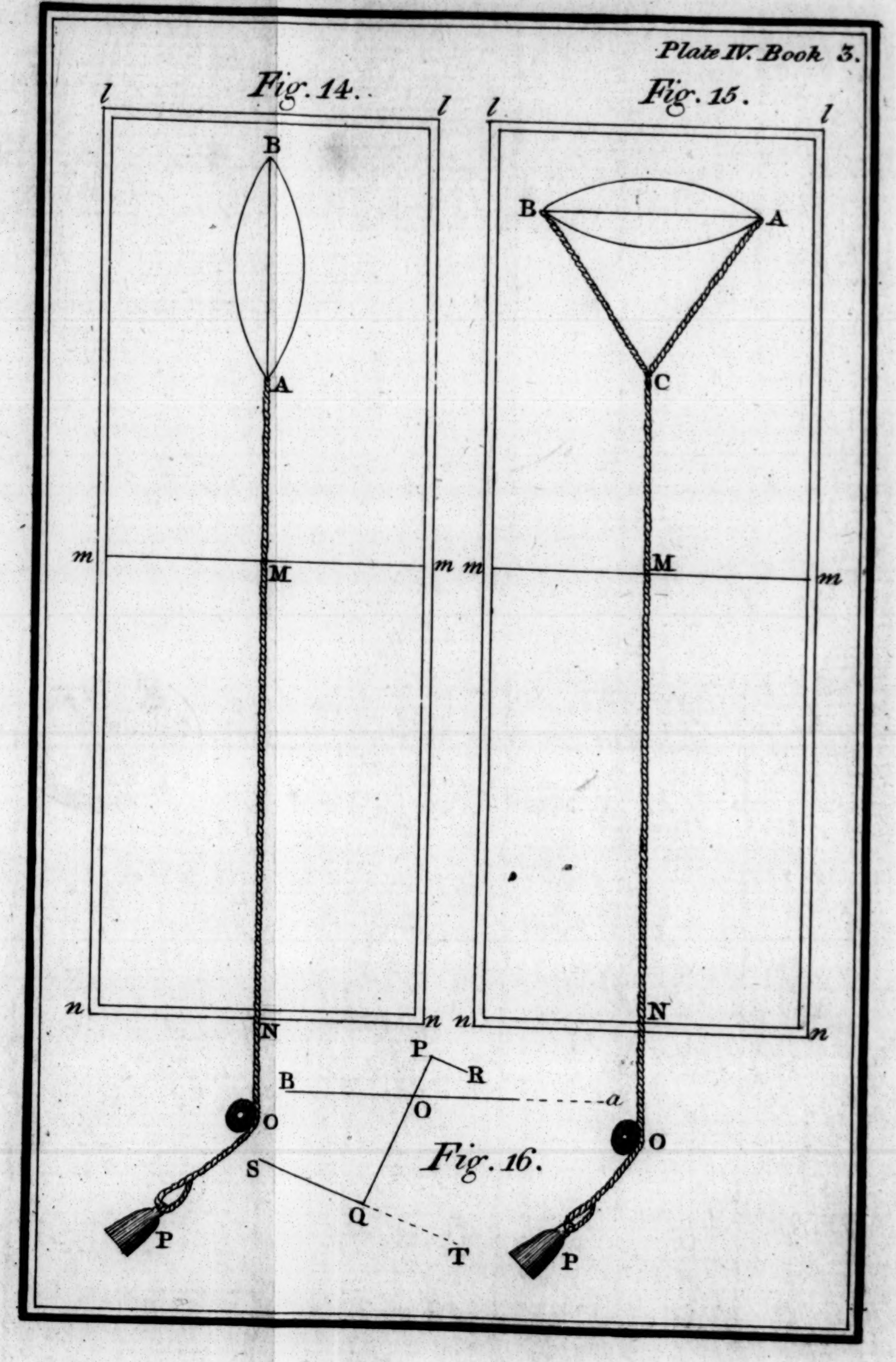


Fig. 14.

Fig. 15.

Fig. 16.

tion of the Parts $\frac{OQ}{OP}$ will then become leſs. These Remarks appear to be abundantly ſufficient for every thing which regards the Action of Oars, and the Motion which will be communicated to the Veſſel.

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